SOME EFFECTS OF EPISTEMOLOGICAL STRUCTURE ON MEMORY

By

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To HSP and WBW

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Major Department: Psychology

The primary concern of this study was the investigation of the effects on memory of the epistemological structure imposed upon a set of concepts by their definitions. A mathematical formalization of this structure was accomplished by relating it to the graph theoretical concept of acyclic digraph. This formalization aided the identification of many structural characteristics of this type of epistemological structure.

A computer program was used to construct each subject's epistemological structuring of 50 concepts from set theory and to ascertain the associated structural characteristics. Central to the approach taken in this study was the assumption that each subject's particular structuring of the set of concepts was of primary importance to his subsequent performance in memory tasks involving these concepts. This approach contrasts greatly with that of other researchers

in the area of semantic memory.

Three graphical characteristics of the subjects' epistemological structures -- the mean distance between concept pairs, the number of paths between concept pairs, and the abstractness of the concepts, i.e., the mean distance from primitive or undefined concepts -- were manipulated in a series of experiments to assess their effect upon recall. Attempts were also made to study the consistency of definitions between and within subjects and to give some indication, by means of discriminant function analysis, of the importance of the nonmanipulated graphical characteristics.

The major result of the experiments reported in this work was that semantic distance as quantified by mean path length between concepts in the graphical representation of a subject's epistemological structure was a potent variable affecting accuracy and confidence of recall in a paired associate task. A corollary of this result was that both accuracy and confidence of recall for response terms of concept pairs connected by paths were decisively better than those of concept pairs not connected by paths. In addition, it was found that there was a high level of agreement of definition within subjects but comparatively low levels of agreement between subjects.

The success of this model of epistemological structure led to the proposal of a more general graphical model. The proposed model would make possible the representation of

epistemological relationships between the genera of concepts, the differentiae of concepts, theorems, and proofs of theorems. The primary purpose of the proposed model is the same as that of the model of epistemological structure employed in these studies: to aid the experimental investigation of the importance of epistemological relationships in complex cognitive activities like those involved in human memory by formulating these relationships in explicit mathematical terms.

CHAPTER I

INTRODUCTION

This study is concerned with some effects on long-term memory of the epistemological structure imposed upon a set of concepts by their definitions. First, I will discuss the epistemological basis of this structure and its relation to the graph theoretic concept of acyclic digraph. Second, I will discuss a computer program which aids the study of this structure. Third, I will describe a number of experiments intended to bring to light some effects of this epistemological structure on memory. Fourth, I will briefly relate this study to the existing literature.

The Epistemological Structure

Epistemological Basis

A concept's definition states the essential characteristics of the concept's referent and thus serves epistemologically to distinguish the referent from everything else. A concept is logically dependent upon the concepts which comprise its definition and these concepts are, in turn, dependent upon the concepts which comprise their definitions, etc. Thus, inherent in any set of concepts is a dependency

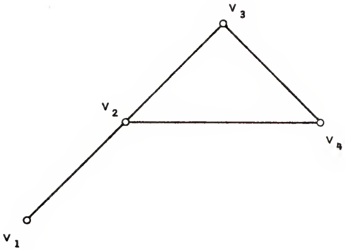
structure imposed upon the concepts by their definitions.

This study is concerned with the effects of this structure on memory. But before specifying the particular investigations which comprise this study, I will describe this epistemological structure in graph theoretic terms. This is done so that the concepts and theorems of graph theory, a branch of mathematics designed to deal with the abstract idea of structure, can be brought into use in these investigations.

Graph Theoretic Description of the Structure

It is necessary to introduce a few concepts from graph theory before attempting to describe this epistemological structure in mathematical terms.

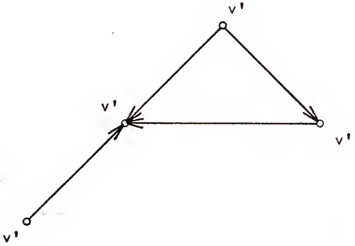
A <u>graph</u> G consists of a finite nonempty set V of p points together with a set E of q edges, unordered pairs of distinct points of V. If the set E contains ordered rather than unordered pairs, then G is said to be a <u>digraph</u> (directed graph). Figure I depicts a graph and a digraph. A <u>walk</u> of a graph is an alternating sequence of points and edges $v_0, e_1, v_1, \ldots, e_n, v_n$. A walk is <u>closed</u> if $v_0 = v_n$ and otherwise is <u>open</u>. If all the points of a walk are distinct then the walk is a <u>path</u>. The <u>length</u> of a walk is the number of occurrences of edges on it. A <u>cycle</u> is a closed walk with at least three distinct points. A graph without any cycles is said to be <u>acyclic</u>. A graph is <u>connected</u> if every pair of points are joined by a path.



$G = (V, E)$

$V = \{v_1, v_2, v_3, v_4\}$

$E = \{(v_1, v_2), (v_2, v_3), (v_3, v_4), (v_2, v_4)\}$



$G' = (V', E')$

$V' = \{v_1', v_2', v_3', v_4'\}$

$E' = \{(v_1', v_2'), (v_3', v_2'), (v_3', v_4'), (v_4', v_2')\}$

Figure 1. A Graph, G, and a Digraph, G'

The _degree_ of a point is the number of edges incident with it. In a digraph, the degree of a point, for the purposes of this study, is specified by the sum-degree of the point. The _sum-degree_ of a point is the sum of the in-degree and out-degree of the point. The _in-degree_ of a point is the number of edges directed into the point, while the _out-degree_ is the number of edges directed out of the point. A _subgraph_ of a graph, G , is a graph whose point set and edge set are subsets of those of G .

The epistemological structure imposed upon a set S of concepts by the relation "concept A is defined in terms of concept B," $\underline{R}$, can be graphically represented as an acyclic digraph in which a point of the graph is associated with each concept of S (S = V) and a directed edge from a given point, v_i , to another point, v_j , exists just in case the concept associated with v_i is defined in terms of the concept associated with v_j .

Figure 2 depicts an acyclic digraph associated with a set of nine concepts symbolized by the letters A through I where concept A is defined in terms of concept B (A $\underline{R}$ B), A $\underline{R}$ C , B $\underline{R}$ E , B $\underline{R}$ D , C $\underline{R}$ D , C $\underline{R}$ F, C $\underline{R}$ I , E $\underline{R}$ G , E $\underline{R}$ H , and D, F, G, H, and I are undefined or primitive concepts.

In order to demonstrate the complexity of the structure imposed by the relation $\underline{R}$ on even a small set of concepts, I have chosen 54 concepts from an introductory chapter of _Graph Theory_ (Harary, 1969) and

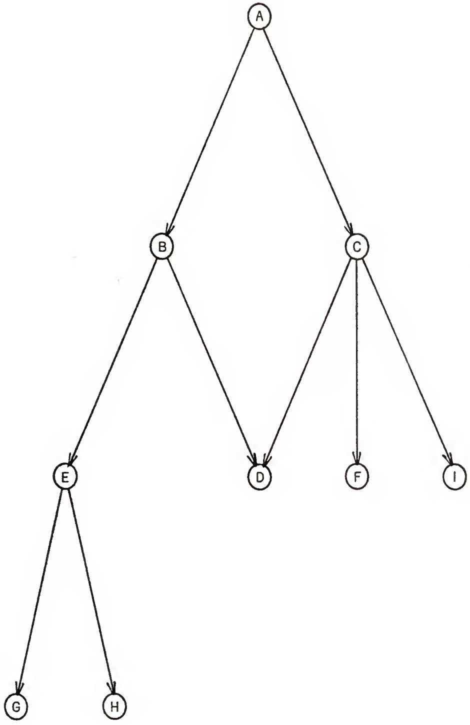


Figure 2. An Acyclic Digraph Associated with a Set of Nine Concepts Symbolized by the Letters A through I

constructed the epistemological structure imposed by their definitions. This structure was constructed by use of the computer program which will be discussed in the next section.

I have included in Appendix A a sixth of the listing of the paths from primitive concepts to nonprimitive concepts and a graph of the associated epistemological structure. The concepts which comprise this structure are indented according to the length of the paths connecting them to primitive concepts. In Figure 3 one can observe the format of the listing in Appendix A from a segment of the structure in which the concept "cycle" is defined in terms of the concept "distinct" ("cycle" $\underline{R}$ "distinct"), "acyclic" $\underline{R}$ "cycle," "tree" $\underline{R}$ "acyclic," "centroid" $\underline{R}$ "tree," "centroid point" $\underline{R}$ "tree," "centroid" $\underline{R}$ "centroid point," "weight" $\underline{R}$ "tree," and "centroid point" $\underline{R}$ "weight." One can also observe in Figure 3 the digraph associated with this segment of the epistemological structure. Note that there are paths to the primitive "distinct" of length 1 from "cycle," of length 2 from "acyclic," of length 3 from "tree," of length 4 from "weight," "centroid point," and "centroid," of length 5 from "centroid" and "centroid point," and of length 6 from "centroid."

By reference to Appendix A one can view some aspects of the structural complexity imposed by the definitions of these 54 concepts; for example, consider that there are 1795 different paths from the nonprimitive concepts to the primitive concepts which range in length from 1 to 10.

DISTINCT

CYCLE

ACYCLIC

TREE

CENTROID

CENTROID POINT

CENTROID

WEIGHT

CENTROID POINT

CENTROID

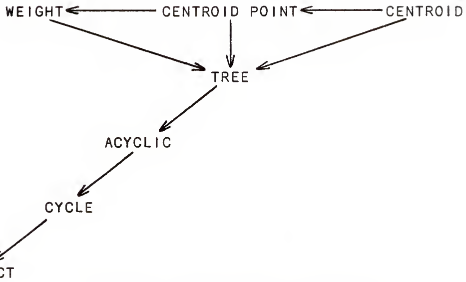


Figure 3. Format of Listing in Appendix A and an Associated Digraph

Because of the complexity of the epistemological structure associated with a set of concepts, any extensive investigation of this structure requires the use of computerized methods of dealing with it. In the next section, I will discuss a computer program which was used in this study.

A Computer Program

The computer program used to construct the epistemological structures of the subjects in this study was originated by Dr. Robert E. Osteen. Although this program was not intended for use in this study, it is a general program for dealing with any data base which can be represented by an acyclic digraph and only minor revisions have been necessary to make it suit the requirements of this study.

The input to the program is a set of concepts, the definitions of the concepts, and the key concepts from the definitions. The program forms the epistemological structure, prints all the paths from the primitive concepts to the other concepts in the format of Figure 3, prints an alphabetical list of the concepts which includes each concept's fathers and sons (i.e., the concepts which this concept is defined in terms of and the concepts which are defined in terms of this concept), the in-degree of the concept, the out-degree of the concept, the concept's full

definition, a list of all the paths from this concept to primitives with their lengths and the mean of these lengths, and the distance between every pair of concepts connected by a path. Appendix A contains some of the output from this program when run on the example data from graph theory and has been discussed previously. I have included in Appendix B a portion of the other information provided by the program from a run with the example data.

Figure 4 illustrates the format of the output in Appendix B. From this figure one can observe the following information about the concept "complete": it is defined in terms of the concepts "point," "adjacent," and "graph" while the concept "clique" is defined in terms of it; the full definition of "complete" is "A graph is complete if every pair of points are adjacent"; that there are paths of length 1 from "complete" to the primitive concepts numbered 3 and 60 in this listing, of length 2 to the primitive concepts numbered 30, 50, 60, 65, and 77; and that the mean length of all these paths to primitive concepts is 1.714.

Thus, by use of this computer program, the epistemological structure associated with a set of concepts can be formed and many of its structural characteristics ascertained. The identification of these characteristics allows one to manipulate them as independent variables by appropriate selection of concepts and thus to study their effect upon memory. In the next section, the specific structural characteristics investigated in this study are discussed and a

17

ELT NAME = COMPLETE

ELT'S PRESENT FATHERS:

POINT

ADJACENT

GRAPH

OUT DEGREE: 3

ELT'S SONS:

CLIQUE

IN DEGREE: 1

ELT DATA:

A GRAPH IS COMPLETE IF EVERY PAIR OF ITS POINTS ARE ADJACENT.

(3,1) (60,1) (50,2) (30,2) (60,2) (65,2) (77,2)

MEAN PATH LENGTH: 1.714

Figure 4. Format of Output Included in Appendix B

general overview of the experiments which comprise this study is given.

Overview of the Experiments

In order to give an overview of the experiments which comprise this study and to facilitate the presentation of these experiments and their results in the next chapter, in this section I will discuss the general strategy of the experiments, the choice of subjects, the construction of a subject's epistemological structuring of a set of concepts, and the specific structural characteristics investigated in this study.

The General Strategy

By formalizing, in graph theoretic terms, the epistemological structure imposed upon a set of concepts by their definitions, it becomes possible to ascertain many structural characterics of this epistemological structure. Once these characteristics have been identified, it is then possible to experimentally investigate their effect upon performance in psychological tasks in which such epistemological information is thought to be important.

The general strategy of the experiments reported in the next chapter is to identify the structural characteristics of a subject's epistemological structuring of a set of concepts and then to vary the concepts presented in memory tasks

in such a way that the effect upon memory recall of these structural characteristics can be assessed.

The Choice of Subjects

I chose to use mathematicians as subjects in this study. This choice was dictated primarily by the fact that it allowed the use of mathematical concepts as the stimulus materials in the experiments. The motivation to use mathematical concepts was that these concepts are, in general, much more precisely formulated than concepts from other areas and, thus, ideally suited to the construction of their associated epistemological structure.

The choice of mathematicians as subjects also allowed the restriction of the set of concepts used in the experiments to a rather limited set, namely, those of naive set theory (Halmos, 1960), with the assurance that since most mathematicians are well acquainted with set theory, the familiarity of the subjects with the set of concepts used was reasonably uniform.

The Construction of a Subject's Epistemological Structure

Fifty concepts were chosen from a text on set theory (Halmos, 1960) and each subject was asked to define these concepts. The epistemological structure imposed on the set of concepts by a subject's definitions of them was constructed by use of the computer program mentioned earlier. Thus, each subject's particular epistemological structuring of the

set of concepts was known along with the structural characteristics associated with each concept as determined by the graphical formulation of the structure.

The Specific Structural Characteristics Investigated

In each of the three experiments discussed in the next chapter, one characteristic of a subject's epistemological structure was manipulated, by appropriate selection of the concepts presented to the subject, and its effect upon memory recall was assessed. The particular characteristics studied were the number of paths between pairs of concepts, the lengths of such paths, and the abstractness of the concepts. For purposes of this study, the abstractness of a concept was defined to be the mean path length of the concept from the primitive concepts. A paired associate paradigm was employed to investigate the number and length of paths between concept pairs while a free recall paradigm was used in the investigation of the abstractness of the concepts.

Although not directly manipulated, an attempt was made to investigate the importance, both relative to each other and independently, of the following characteristics on recall:

(a) the in-degree, out-degree, and sum-degree of the stimulus and response concepts.

(b) the number, minimum length, and maximum length of paths to primitive concepts from stimulus and response concepts.

(c) the minimum and maximum path length between pairs of concepts.

An attempt was also made to determine the consistency of the subjects' epistemological structurings of the set of concepts.

Relation of This Study to the Existing Literature

The study of human memory is presently one of the fastest growing research areas in experimental psychology. In the last decade, while the number of publications in experimental psychology increased by a factor of about 2.5, those in the field of human memory increased by a factor of close to 6.0 (Tulving and Donaldson, 1972). This rapid growth is supplemented by the work in computer science concerned with semantic information processing (Minsky, 1968). One reason that can be put forth to account for this rapid growth is the burgeoning interest in the study of the organization or structure of memory in general (Bower, 1970, 1972; Anderson, 1972; Anderson and Bower, 1973; Tulving and Donaldson, 1972; Kintsch, 1970; Norman, 1970) and of semantic memory in particular (Quillian, 1968, 1969, 1972).

Since the present investigation is more related to work in the comparatively new area of semantic memory than it is to traditional memory research, I will, in an effort to relate this study to the existing literature, briefly

discuss the category of semantic memory and give a synopsis of the research in this area and contrast the approach of this study to that of other work in this area.

Semantic Memory

The term "semantic memory" was introduced by Quillian (1969) to refer to the memory organization he designed for his computer program, the Teachable Language Comprehender (TLC). This organization of memory was intended to make it possible for TLC to make humanlike use of semantic information.

In a recent article Tulving (1972) contends that it often aids conceptual precision to divide a general category such as memory into complementary subordinate categories such as short-term and long-term memory. Tulving's thesis, in this article, is that a conceptually useful division of the category of memory can also be made by differentiating semantic memory from a complementary category for which he introduces the term episodic memory. Following Tulving's analysis for the most part, I will present, in an effort to map out what is meant by semantic memory, characterizations of semantic memory and of episodic memory, the form of memory semantic memory is not.

Quillian (1969) conceptualizes semantic memory as the form of memory used in handling nonemotive aspects of a concept's meaning rather than the emotive aspects such as Osgood's (1957) semantic differential is concerned with.

Tulving (1972) presents a quite clear sketch of semantic memory as follows:

> Semantic memory is the memory necessary for the use of language. It is a mental thesaurus, organized knowledge a person possesses about words and other verbal symbols, their meaning and referents, about relations among them, and about rules, formulas, and algorithms for the manipulation of these symbols, concepts, and relations. (Tulving, 1972, p. 386)

On the other hand, Tulving characterizes episodic or nonsemantic memory as memory which "receives and stores information about temporally dated episodes or events, and temporal-spatial relations among these events " (Tulving, 1972, p. 385).

Before turning to a brief synopsis of the literature of semantic memory, the graphical format of Quillian's model of semantic memory will be presented.

Quillian's (1969) model of semantic memory contains specifications of two kinds of information: factual statements and syntactic information which is specified by form-tests. The format of the memory structure is intended to allow the "representation of everything uniformly enough to be dealt with by specifiable procedures, while being rich enough to allow encoding of natural language without loss of information " (Quillian, 1969, p.462).

The factual information contained in memory is encoded as either a unit or a property. A unit is the representation of a concept and a property is the means of encoding predications which serve to modify and refine the meaning of a unit or another property.

The general format of a unit and a property is given In Figure 5. From this figure, one can observe that a unit contains pointers (in graphical terminology, directed edges) to another unit which serves as the unit's superset, in Quillian's terminology, but which might be better characterized as the genus of the concept specified by the unit and any number of pointers to properties which refine the meaning of the unit. One can observe in Figure 5 that a property contains pointers to its attribute, the value of its attribute, and to any number of properties which refine its meaning. Examples of attributes are verbs and prepositions and, in this case, their values would be their grammatical objects. In order to clarify the ideas of units and properties, I have depicted in Figure 6 the encoding of the concept "intellectual" when defined as follows: An intellectual is a person who pursues truth by reasoning. Here one can see that the superset of the unit "intellectual" is the unit "person," that the former unit is modified by the property "pursues truth," and that this property is further modified by the property "by reasoning." Also presented in Figure 6 is the representation of this structure by a digraph. In the digraph a unit is represented by either the word which stands for the unit or the symbol "□" and a property is represented by the symbol "O".

One should note the differences between the graphical structure put forth by Quillian and that of the epistemological structure with which this study is concerned. Quillian's

[pointer to unit's superset, pointers to properties]

<u>A UNIT</u>

(pointer to attribute, pointer to value, pointers to properties)

<u>A PROPERTY</u>

Figure 5. The General Format of a Unit and a Property

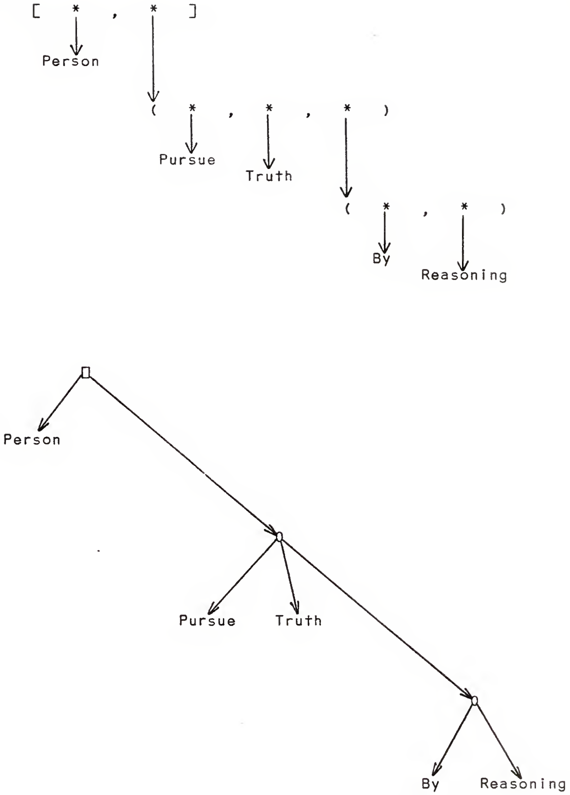


Figure 6. The Encoding of a Concept
and an Associated Digraph

graphical representation allows for the encoding of numerous types of relationships between concepts while this study, because of its focus, represents only one type of relationship: that of definition, i.e., that one concept is defined in terms of another.

Synopsis of Semantic Memory Research

As Tulving and Donaldson (1972) have pointed out, the major portion of the work in the area of semantic memory has been theoretical rather than experimental. This theoretical work has been primarily involved with the formulation of models of semantic memory (Quillian, 1969; Collins and Quillian, 1969, 1970a, 1970b; Meyer, 1970; Kintsch, 1970; Shaeffer and Wallace, 1970; Rumelhart, Lindsay, and Norman, 1972; Anderson, 1972; Anderson and Bower, 1973).

In the experimental work dealing with semantic memory, most investigators have followed Collins and Quillian (1969) in using reaction time tasks to study the validity of certain aspects of the proposed models of semantic memory. The general format of these tasks has been to present subjects with either pairs of concepts or sentences relating pairs of concepts of the form "An S is a P" and measure the time required for the subjects to make some semantic decision. Quillian's model of semantic memory is based on a rather strict assumption of economy of storage. For example, it is assumed that a property is stored only with the most general unit to which the property applies. Thus, Quillian's model predicts

that subjects will take "less time to confirm a sentence like 'A collie is a dog' than to confirm a sentence like 'A collie is an animal'" (Collins and Quillian, 1970a, p. 432). This prediction is based on the greater semantic distance between collie and animal than between collie and dog since the former requires chaining two units along the superset chain from collie while the latter requires chaining only one unit from collie. Collins and Quillian in a number of studies (1969, 1970a, 1970b) have found reaction time for verification of this type of sentence to be an increasing function of semantic distance. Others have argued that the crucial variable in these studies was not semantic distance as specified in Quillian's model but either the associative frequency of the concepts (Conrad, 1972) or the size of the concept categories (Wilkins, 1971; Freedman and Loftus, 1971).

In these studies the relationship imposing the semantic distance measure between pairs of concepts is usually that of set inclusion, i.e., the superset relation, between hierarchically organized taxonomic categories (e.g., animals, mammals, birds). The present study approaches the experimental investigation of semantic memory with a different strategy from that of the reaction time studies. In the next section, the strategy of this study is contrasted with that of previous work in this area.

This Study's Strategy Contrasted with That of Previous Work

The present study differs in four rather fundamental ways from other experimental work in the area of semantic memory. First and most importantly, while previous studies have always assumed some one structuring of a set of concepts for all subjects participating in an experiment, the present study, in an effort to increase experimental control, did not make such an assumption but rather constructed each subject's own epistemological structuring of the set of concepts. Second, the particular relationship imposing the structure was that of definition rather than set inclusion as in most previous work. The relationship of definition being a more general relationship than that of set inclusion because it includes both genus and differentia type relations between concepts while that of set inclusion or superset relationship, in Quillian's terminology, is in essence, as I have argued, just that of the genus relationship. Third, the task employed was one of memory recall rather than one involving reaction time. Fourth, while previous research has dealt almost exclusively with semantic distance, the present study in addition to dealing with semantic distance, although from a different approach, has also attempted to assess the effects of numerous other graphical characteristics of the epistemological structure on recall.

CHAPTER II

THE EXPERIMENTS

In this chapter, the methods, results and discussions of the experiments which comprise this study are presented. The first three experiments involved assessments of the effect of some graphical characteristics of the _S_s' epistemological structurings of the set of concepts on recall. The graphical characteristics manipulated in the first three experiments were the distance between pairs of concepts, the number of paths between pairs of concepts, and the mean path length of concepts from primitive concepts. The results of attempts to assess the effect on recall of the graphical characteristics not directly manipulated as well as the consistency of the _S_s' epistemological structurings of the set of concepts are also presented.

Experiment I: Method

Subjects

Ten graduate students and faculty members of the University of Florida department of mathematics participated in the experiment. Each _S_ was paid for his participation in this and the following experiments.

Design

For each S 2 pairs of concepts were formed based upon his constructed epistemological structure. A given pair of concepts was connected in the graphical representation of a S's epistemological structure by a short path (mean length of 1.06 and standard deviation of 0.23), a long path (mean length of 3.86 and standard deviation of 0.69), or was not connected by any path. A within-S design was employed crossing distance between concepts with trials.

Procedure

For each S 5 random orders of the S's 20 pairs of concepts were constructed with the restriction that a proportionate number of the 3 types of pairings occurred in each quarter of an ordering (see Appendix D for a sample list). The 20 pairs of concepts were presented tachistoscopically to a S at the rate of 1 pair every 8 sec. with a pair visible for 2 sec. After exposure to the 20 pairs of concepts, a S participated in a decoding task for 2 min. to preclude recall from short-term memory (see Appendix E). A S was then given a sheet of paper containing all of the left-hand members of the pairs of concepts and asked to write next to each concept the associated right-hand member and to give an assessment of his confidence on a 4-point scale. Four such study-test trials were completed in an experimental session. After a period of 24 hours, each S returned and was given a test trial followed by a study-test trial.

Experiment I: Results

Analysis of Mean Percentage Correct Recall

The mean percentage correct recall of the response terms (MPCRT) for each distance condition was computed for each *S* on each trial. Table I contains a summary of a 2-factor repeated measures analysis of variance performed on this measure. The overall effects of distance, of trials, and of the interaction between distance and trials were significant ($p < .001$). Figure 7 depicts the MPCRT on the 6 trials for each distance condition.

Tukey's HSD test (Kirk, 1968) was used to make all pairwise comparisons between the means of the three levels of the distance condition and between the means of the 6 trials. Table 2 contains the results of this analysis. Because of the significant interaction between distance and trials and because the distance variable was of primary interest in this experiment, Tukey's HSD test was used to compare the differences between the means of each level of the distance variable on each trial. This analysis revealed, as can be seen in Table 3, that on the first two trials the MPCRT of concept pairs connected by short, long, or no paths was significantly ($p < .01$) different from each other with recall highest for short pairs, next highest for long pairs, and lowest for unconnected pairs; that MPCRT of short pairings was significantly ($p < .05$) larger than those for

TABLE I

SUMMARY OF ANALYSIS OF VARIANCE OF THE MEAN PERCENTAGE OF CORRECT RECALL

	DF	SS	MS	F
Distance (D)	2	1.967594	0.9837971	39.4317 *
Trials (T)	5	5.030701	1.0061400	98.3321 *
Subjects(D)	18	0.4490891	0.0249494	
Subjects(T)	45	0.4604425	0.0102305	
Distance × Trials	10	1.0282360	0.1028236	13.1671 *
Subjects(D × T)	90	0.7028236	0.0078092	

* $p < .001$

TABLE 2

RESULTS OF TUKEY'S HSD TEST PERFORMED ON THE MPCRT FOR DISTANCE AND TRIALS

Distance Means		
Short	Long	None
94.066	76.483	69.150

Trial Means					
T1	T2	T3	T4	T5	T6
49.833	65.033	87.100	92.433	88.933	96.066

Differences Between Means	
Distance	
Short - Long	17.583
Short - None	24.916*
Long - None	7.333

* p < .05

Differences Between Means		
Trials		
T1 - T2 15.20*	T1 - T3 37.27**	T1 - T4 42.60**
T1 - T5 39.10**	T1 - T6 46.23**	T2 - T3 22.07**
T2 - T4 27.40**	T2 - T5 24.30**	T2 - T6 31.03**
T3 - T4 5.33	T3 - T5 1.83	T3 - T6 8.97
T4 - T5 -3.50	T4 - T6 3.63	T5 - T6 7.13

* p < .05

** p < .01

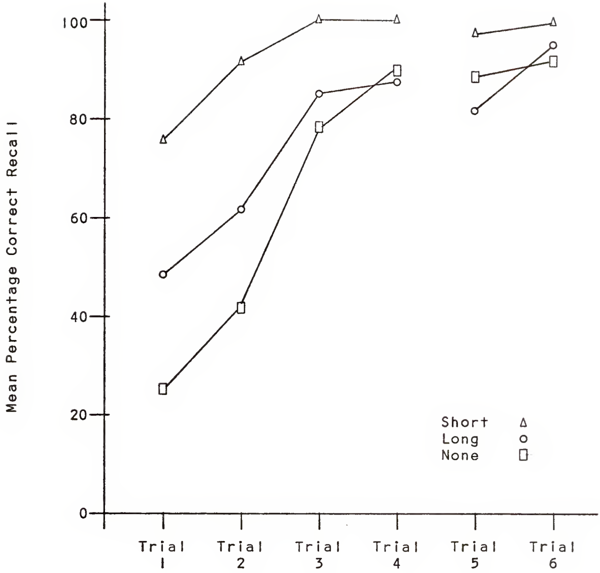


Figure 7. Mean Percentage Correct Recall on Each of the Six Trials for the Three Distance Conditions

TABLE 3

DIFFERENCES BETWEEN MPCRT FOR THE DISTANCE CONDITIONS ON EACH TRIAL

	T1	T2	T3	T4	T5	T6
Short - Long	27.8**	29.9**	15.8*	11.7	16.0*	4.3
Short - None	50.7**	48.3**	22.9**	11.0	9.1	7.5
Long - None	22.9**	18.4**	7.1	0.7	6.9	3.2

* $p < .05$

** $p < .01$

long pairings on trials 3 and 5; and that the MPCRT for short pairs were significantly ($p < .01$) greater than for pairs not connected by a path on trial 3.

Analysis of Mean Confidence Rating

For each trial, each S's mean confidence rating for recall of the response terms in each distance condition was computed. A summary of a 2-factor repeated measures analysis of variance performed on this measure is given in Table 4. As with the analysis of MPCRT, the effects of distance, of trials, and of their interaction were all significant ($p < .001$). Figure 8 shows the mean confidence rating on the 6 trials for each type of concept pairing.

Tukey's HSD test was used to make all pairwise comparisons between the means of the 3 levels of the distance condition, between the means of the 6 trials, and between the means of each level of the distance variable on each trial. As one can observe from Tables 5 and 6, the results almost exactly parallel those of the MPCRT analysis. On trials 1 and 2, Ss were significantly ($p < .01$ or $p < .05$) more confident of their recall of response terms of concept pairs connected by short paths than those not connected by any paths or those connected by long paths, and the confidence of recall of response terms of long pairings was significantly ($p < .01$) higher than that of those not connected by paths. Other significant ($p < .01$) differences were found on trials 3 and 5 with confidence of recall for the short pairings

TABLE 4

SUMMARY OF ANALYSIS OF VARIANCE OF THE
MEAN CONFIDENCE RATING

	DF	SS	MS	F
Distance (D)	2	20.02376	10.01188	19.0415 *
Trials (T)	5	48.68991	9.73798	53.5863 *
Subjects(D)	18	9.46424	0.52579	
Subjects(T)	45	8.17763	0.18172	
Distance × Trials	10	11.94589	1.19459	9.3416 *
Subjects(D × T)	90	11.50911	0.12788	

* $p < .001$

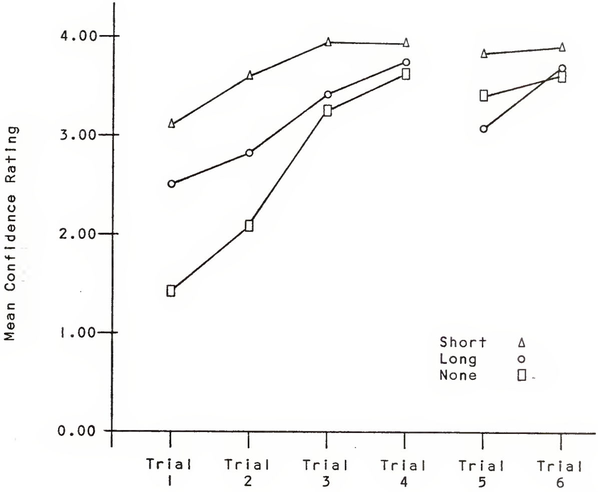


Figure 8. Mean Confidence Rating on Each of the Six Trials for the Three Distance Conditions

TABLE 5

RESULTS OF TUKEY'S HSD TEST PERFORMED ON THE MEAN CONFIDENCE RATINGS FOR DISTANCE AND TRIALS

Distance Means		
Short	Long	None
3.736	3.218	2.930

Trial Means					
T1	T2	T3	T4	T5	T6
2.361	2.850	3.568	3.771	3.447	3.772

Differences Between Means Distance	
Short - Long	.518
Short - None	.806*
Long - None	.288

* p < .05

Differences Between Means Trials		
T1 - T2 .488	T1 - T3 1.027**	T1 - T4 1.410**
T1 - T5 1.086**	T1 - T6 1.411**	T2 - T3 .719**
T2 - T4 .922**	T2 - T5 .598**	T2 - T6 .923**
T3 - T4 .203	T3 - T5 -.121	T3 - T6 .204
T4 - T5 -.324	T4 - T6 .001	T5 - T6 .325

** p < .01

TABLE 6

DIFFERENCES BETWEEN THE MEAN CONFIDENCE RATINGS
FOR THE DISTANCE CONDITIONS ON EACH TRIAL

	T1	T2	T3	T4	T5	T6
Short - Long	0.59 *	0.76 **	0.52	0.27	0.75 **	0.21
Short - None	1.45 **	1.51 **	0.68 *	0.31	0.40	0.27
Long - None	1.07 **	0.75 **	0.16	0.05	-0.35	0.06

* $p < .05$

** $p < .01$

greater than that for long pairings on trial 5 and for those pairings not connected by any paths on trial 3.

Experiment I: Discussion

The effect upon recall of three different levels of semantic relatedness of concept pairs was experimentally investigated using a new approach which was designed to make possible greater control of this variable. In a paired associate task, pairs of concepts which were connected by either long paths, short paths, or were not connected by any paths were employed. Semantic distance, as quantified by mean path length between pairs of concepts in the graphical representation of the epistemological structure a S imposed on a set of concepts by his definition of them, was found to be a potent variable affecting memory.

The results were in accordance with a straightforward extrapolation from reaction time studies of semantic memory: recall and confidence of recall were highest for response terms of concept pairs connected by short paths, next highest for those of concept pairs connected by long paths, and lowest for response terms of concept pairs not connected by any paths. Although this ordering of recall was found only on the initial trials, parallel results were obtained from the analysis of the two dependent variables: mean percentage correct response and mean confidence rating.

While there was no significant difference between either recall or confidence of recall for response terms of

concept pairs not connected by paths and those of concept pairs connected by long paths subsequent to the second trial, it is interesting to note that as late as the fifth trial, i.e., the first delayed trial, there was a significant difference for recall and confidence of recall for response terms of concept pairs connected by short paths and those of concept pairs connected by long paths. One can reasonably postulate, if retrieval strategies make use of the structural aspects of the epistemological structure, that Ss adopted different strategies for recall of response terms of concept pairs connected by paths and those not connected by paths. Based on this assumption, the results indicate that the strategy for pairs connected by paths was, in accordance with reaction time studies, more effective, even as late as the fifth trial, for those pairs connected by short paths while the strategy for unconnected pairs was, subsequent to the third trial, an equally effective recall strategy. It can, on the other hand, be argued that the failure of a continuation of the clear distinction between recall for the three different types of pairings, following the second trial, was due to nothing more than a ceiling effect of approaching perfect recall. In any case, the results of the analyses of recall and confidence of recall for response terms of concept pairs connected by paths are in agreement with Quillian's model of semantic memory and contribute evidence for semantic distance being a variable of importance in memory tasks involving concepts.

In regard to any conclusions concerning the relative effectiveness of the retrieval strategy used with concepts connected by paths and the strategy used with concepts not connected by paths, if one accepts the assumption of different strategies, one point should be made clear. The former strategy because of its dependence on the meanings, i.e., the definitions, of the concepts is one that would readily be adopted by a S since it is the meaning of concepts which the S normally deals with. If this is in fact the case, the inferiority of recall, on the initial trials, for response terms of concept pairs not connected by paths could be accounted for by the S's initially attempting to adopt this type of strategy and because of the lack of such relationships between the concepts, as evidenced by the graphical representation, finding it unsuccessful and then turning to some other recall strategy based on other aspects of the concepts.

Whether or not different recall strategies were employed by Ss in dealing with the different types of pairs, it seems clear that the graphical representation of the epistemological structure a S imposes on a set of concepts by his definitions of them has led to the quantitative identification of a relationship between concepts, semantic distance, which is an important variable in the type of memory tasks used in this experiment.

Experiment 2: Method

Subjects

The same 10 graduate students and faculty members of the department of mathematics served as Ss in this experiment.

Design

For each S 20 pairs of concepts were formed based on his constructed epistemological structure. A given pair of concepts was connected in the graphical representation of a S's epistemological structure by a small number of paths (mean of 1.12 and standard deviation of 0.33), by a large number of paths (mean 6.97 and standard deviation of 3.95), or by no paths. An attempt was made to hold the distance between concepts connected by paths constant. A within-S design was employed crossing the two factors: number of paths between concepts and trials.

Procedure

For each S 4 random orders of the S's 20 pairs of concepts were constructed with the restriction that a proportionate number of the 3 types occurred in each quarter of an ordering (see Appendix D for a sample list). The 20 pairs of concepts were presented tachistoscopically to a S at the rate of 1 pair every 8 sec. with a pair visible for 2 sec. After exposure to the 20 pairs of concepts, a S participated in a decoding task (see Appendix E) for 2 min. to preclude

recall from short-term memory. A S was then given a sheet of paper containing all of the left-hand members of the pairs of concepts and asked to write next to each concept the associated right-hand member of the pair and to give an assessment of his confidence on a 4-point scale. Four such study-test trials were completed in the experimental session.

Experiment 2: Results

Analysis of Mean Percentage Correct Recall

The MPCRT for each level of the number of paths variable was computed for each S on each trial. A 2-factor repeated measures analysis of variance revealed a significant ($p < .01$) overall effect of number of paths, trials, and a significant ($p < .05$) interaction between trials and number of paths. Figure 9 depicts the MPCRT for the four trials for each type of concept pairing.

As in the first experiment, Tukey's HSD test was employed to make all pairwise comparisons between the MPCRT of the three levels of the path condition, between the MPCRT of the four trials, and between the MPCRT for each number of paths on each trial. It was found that there was no significant ($p < .05$) difference, either overall or on any trial, between MPCRT for concept pairs connected by a small number of paths and those connected by a large number of paths. It was also found that, both overall and on the first three

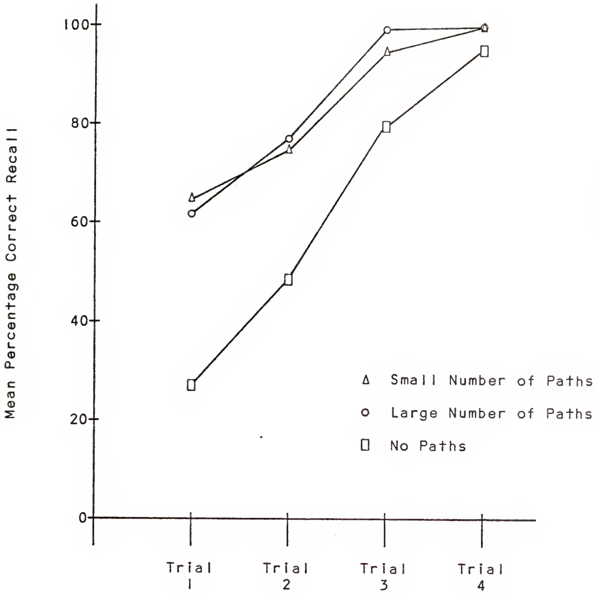


Figure 9. Mean Percentage Correct Recall on Each of the Four Trials for the Three Levels of the Path Condition

trials, the MPCRT for pairs of concepts not connected by paths was significantly ($p < .05$) less than those connected by paths.

Analysis of Mean Confidence Rating

For each trial, a S's mean confidence rating for recall of the response terms of the three types of concept pairings was computed. Figure 10 shows the mean confidence ratings on the four trials for each type of concept pairing. The results of an analysis of variance performed on these mean confidence ratings, as in the first experiment, paralleled those of the MPCRT analysis. The effects of the number of paths, trials, and the interaction between number of paths and trials were significant ($p < .01$).

Tukey's HSD test revealed that there was no significant ($p > .05$) difference, either overall or on any trial, between the mean confidence rating for recall of the response terms of concepts connected by a small number of paths and those of concepts connected by a large number of paths. As with the MPCRT analysis, the mean confidence ratings of Ss were significantly ($p < .05$) higher for recall of response terms of concept pairs connected by either a small or large number of paths than for those not connected by any paths, both overall and on the first two trials.

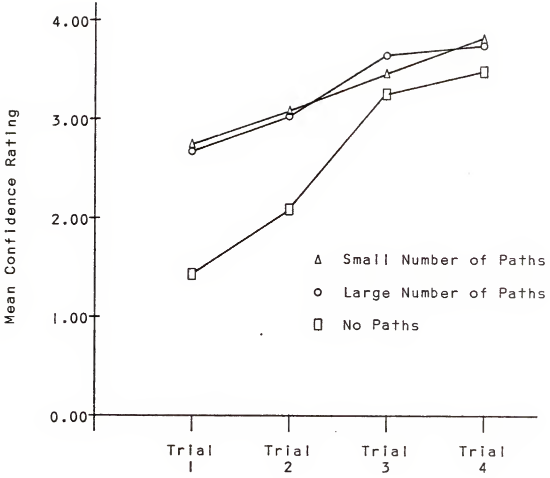


Figure 10. Mean Confidence Rating on Each of the Four Trials for the Three Types of Concept Pairings

Experiment 2: Discussion

In this experiment, no differences were found for recall or confidence of recall of response terms of concept pairs connected by many paths and those connected by few paths. As in the first experiment though, there was, on the initial trials, a significant superiority of recall and confidence of recall for response terms of concept pairs which were connected by paths over those of pairs not connected by paths. The results of the analyses of the two dependent variables again were parallel.

The strength of associative relationships between word pairs has been shown (see for example, Jenkins, Mink, and Russel, 1958) to be an important variable affecting recall in paired associate tasks. One measure of strength of association devised by Deese (1962) is simply to count the number of associates a pair of words have in common.

There are some grounds for postulating that concept pairs with many paths between them would have more common associates than those with only a few connecting paths. Observe that in Figure II, the concepts E and G are associated with concepts H, F, and I while concept A and C are only associated with concept B. This assumption would lead one to expect that recall of response terms of pairs connected by many paths would be superior to that of pairs connected by a few paths. The results of this experiment are not in accordance with this prediction.

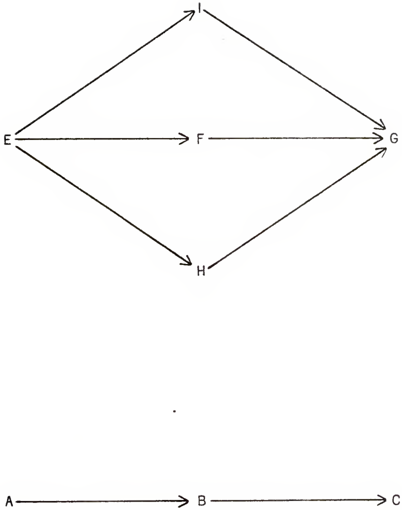


Figure 11. Pairs of Concepts Differing in the Number of Paths Between Them

It should be pointed out that the results of this experiment are in agreement with one aspect of Quillian's theory of semantic memory. Quillian (1969) contends that search procedures dealing with semantic memory proceed in parallel fanning out from a given concept in a "breadth-first" fashion. In terms of the graphical representation presented in this study, this type of search procedure would proceed along each path joining a pair of concepts in parallel. Therefore, the number of paths between a pair of concepts would not be a factor influencing recall.

Experiment 3: Method

Subjects

The same 10 graduate students and faculty members of the department of mathematics served as Ss.

Design

For each S, based on his epistemological structuring of the set of concepts, 2 nonintersecting sets of 10 concepts each were identified such that the difference in mean path length from primitive concepts was maximized between the 2 sets. Across Ss, the mean of the mean path lengths from primitive concepts for one set was 0.95 with standard deviation of 0.36 and for the other the mean was 3.78 with standard deviation of 0.43. The former set of concepts will be termed "concrete" while the latter will be called

"abstract." A free recall experiment employing a within-S design was used to ascertain the effects of abstractness and concreteness across 4 trials.

Procedure

For each S 4 random lists of each S's 20 concepts were constructed with the restriction that a proportionate number of concrete and abstract concepts occurred in each quarter of a list (see Appendix D for a sample list).

On each of 4 trials, a S was allowed to view 1 of his 4 lists for a period of 11 sec. Following each trial, a S participated in a decoding task (see Appendix E) for 2 min. to preclude recall from short-term memory. After the decoding task on each trial, a S was given a sheet of paper and asked to write all of the concepts which had been presented and next to each give an assessment of his confidence of correct recall on a 4-point scale.

Experiment 3: Results

For each S the mean percentage of correct recall and mean confidence ratings for recall of abstract and concrete concepts on each trial were computed. A 2-factor repeated measures analysis of variance, crossing the abstractness of the concepts with trials, was used to analyze these data.

The analyses revealed no significant ($F < 1$) difference between accuracy or confidence of recall for

abstract and concrete concepts. Figure 12 depicts the means for these two dependent variables on each trial for concrete and abstract concepts. There was no significant interaction ($\underline{F} < 1$) between the abstractness of the concepts and trials but the usual significant ($p < .01$) trials effect was found.

In order to quantify the extent of clustering of concrete and abstract concepts in the Ss' recall, the ARC (Roenker, Thompson, and Brown, 1971) index of clustering was computed from the recall of each S on each trial. The ARC score is a measure of clustering which sets chance clustering at zero and perfect clustering at one. The score is obtained by dividing the actual number of category repetitions above chance by the maximum possible number of such repetitions above chance. The exact formula used to calculate the ARC scores is given in Appendix F.

Table 8 shows the mean and standard deviations of the ARC scores for the Ss on each trial. A single factor repeated measures analysis of variance revealed no significant ($\underline{F} < 1$) clustering differences between trials.

Experiment 3: Discussion

In this experiment, abstract and concrete concepts were presented in randomized lists using a free recall paradigm. There were no significant differences ($\underline{F} < 1$) for recall or confidence of recall between the two types of concepts.

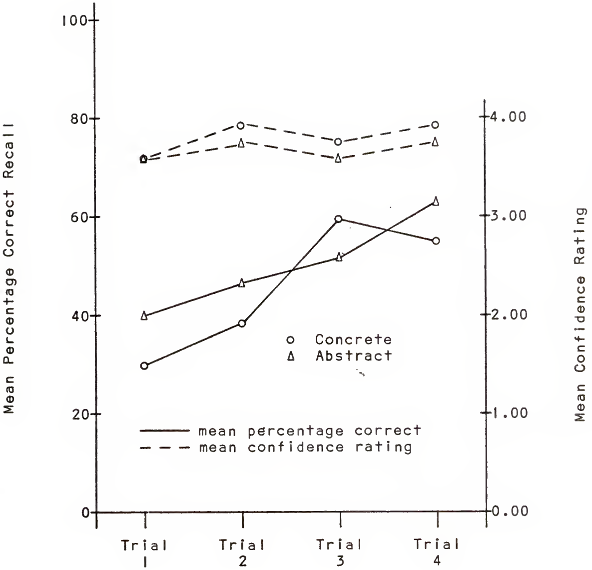


Figure 12. Mean Percentage Correct and Mean Confidence Rating for Recall of Abstract and Concrete Concepts

TABLE 7

MEANS AND STANDARD DEVIATIONS
OF ARC SCORES ON EACH TRIAL

	Trial 1	Trial 2	Trial 3	Trial 4
Mean	.372	.131	.123	.197
Standard Deviation	.683	.515	.327	.270

The abstractness of a given concept was quantified by the mean path length between the concept and the primitive concepts to which it was connected in the graphical representation of a S's epistemological structuring of the set of concepts. Thus, this quantification of the abstractness of a concept is dependent upon the definition of the concept.

One can speculate that the recall strategies employed by Ss attempting to recall a list of words primarily make use of non-defining characteristics of the concepts. If this were the case, one would not expect to observe any difference between recall of abstract and concrete concepts. Following from this speculation, the most fruitful paradigm for assessing possible differential effects of the abstractness variable would be one which focused the Ss recall strategies on the meanings of the concepts. It is conjectured that a paired-associate paradigm would be superior to a list paradigm in this respect and that a paradigm relating the concepts in sentences would be best.

The ARC clustering scores were also in accordance with the above speculation about recall strategy. If the strategies employed by the Ss were dependent upon the factors which distinguished the two types of concepts, one would expect an increase of clustering across trials. This was not the case.

Some Other Analyses

In this section, the results of a stepwise discriminant function analysis of the recall data from the first experiment as well as some assessments of the consistency, both between and within Ss, of the Ss' epistemological structurings of the set of concepts are presented.

Stepwise Discriminant Function Analysis

An attempt was made to assess the effects upon recall in the first experiment of the graphical characteristics which were not directly manipulated in that experiment. A stepwise discriminant function analysis was used in the endeavor to give some indication of the effects upon recall of the nonmanipulated structural variables in relation to the manipulated variable, mean distance between concept pairs. In Appendix G, a brief general description of stepwise discriminant function analysis is given.

Application of discriminant analysis

BMD07M, a computer program described in Appendix G, was used to generate a discriminant function from the data of each recall trial. On each trial the criterion groups were two disjoint collections of concept pairs: those in which the response term was recalled on that trial and those in which the response term was not recalled. The structural

variables which served as the predictor variables for these functions are given in Table 8.

The reasoning behind the application of discriminant analysis was that some indication of the importance of the nonmanipulated characteristics relative to the manipulated one could be assessed by the relative importance of these variables in the discriminant function and by the classification accuracy of this function.

Tables 9-14 present the results of the discriminant analyses for the six trials and Table 15 shows the classification accuracy of the discriminant functions. These results indicate that, on the first three trials, mean distance between concepts was by far the structural variable of greatest importance in the discriminant function. One can observe that on these trials once the variance accounted for by this variable was partialed out no other variable's $\underline{F}$ to remove reached significance and that there was little improvement in the classification accuracy of the discriminant function with the inclusion of additional variables. On the first trial, though, three other distance-related variables were shown to be important predictor variables: minimum distance between concepts, maximum distance between concepts, and abstractness of the stimulus concept.

Subsequent to the third trial, the independent predictor variables of importance in the discriminant function were: on the fourth trial, the number of paths to primitives from the response concept and, on the sixth

TABLE 8

STRUCTURAL CHARACTERISTICS OF THE CONCEPT PAIRS

Variable Number	Structural Characteristic
1	In-degree of the stimulus concept
2	Out-degree of the stimulus concept
3	Sum-degree of the stimulus concept
4	Abstractness of the stimulus concept
5	Number of paths to primitives from the stimulus concept
6	Minimum length of paths to primitives from the stimulus concept
7	Maximum length of paths to primitives from the stimulus concept
8	In-degree of the response concept
9	Out-degree of the response concept
10	Sum-degree of the response concept
11	Abstractness of the response concept
12	Number of paths to primitives from the response concept
13	Minimum length of paths to primitives from the response concept
14	Maximum length of paths to primitives from the response concept
15	Number of paths between the concept pair
16	Mean distance between the concept pair
17	Minimum distance between the concept pair
18	Maximum distance between the concept pair

TABLE 9

RESULTS OF DISCRIMINANT ANALYSIS OF TRIAL I DATA

Predictor Variable	Rank	Independent *F*	*F* to Remove
16	1	22.4095**	22.4095**
15	2	3.9009	1.5243
17	3	4.6591*	2.4427
3	4	0.1012	1.6243
4	5	5.2445*	2.5817
7	6	3.7269	1.3692
6	7	0.3075	0.9947
5	8	1.2083	0.8513
12	9	0.1196	0.5831
14	10	1.0595	0.9455
8	11	3.8491	0.6931
13	12	2.2364	0.1907
10	13	3.9200	0.2015
9	14	1.0092	3.6473
2	15	0.2803	0.6000
18	16	5.1531*	0.3692
11	17	1.0543	0.9680

* $p < .05$

** $p < .01$

TABLE 10

RESULTS OF DISCRIMINANT ANALYSIS OF TRIAL 2 DATA

Predictor Variable	Rank	Independent F	F to Remove
16	1	18.5440**	18.5440**
6	2	3.8176	1.6681
17	3	2.5550	2.0764
2	4	0.0381	2.3374
18	5	3.1306	1.5563
8	6	2.4692	1.3756
12	7	0.0469	1.0705
14	8	0.1764	1.6025
10	9	2.9670	1.2197
9	10	0.0099	0.8520
13	11	2.3015	0.7921
11	12	0.1925	0.5424
5	13	0.0666	0.2969
14	14	0.1764	0.0906
4	15	1.7008	0.0791
7	16	0.9637	0.0227
5	17	0.0666	0.0037
3	18	0.2493	0.0312

**p <.01

TABLE II

RESULTS OF DISCRIMINANT ANALYSIS OF TRIAL 3 DATA

Predictor Variable	Rank	Independent F	F to Remove
16	1	8.1570**	8.1570**
1	2	2.1363	0.7873
13	3	1.8745	0.9692
8	4	1.0472	1.3858
5	5	0.0175	0.4968
4	6	1.8735	2.5057
7	7	1.5761	0.9103
6	8	0.8372	0.4205
14	9	0.0138	0.4094
11	10	0.0202	0.2485
17	11	1.1993	0.2259
15	12	0.2921	0.0672
3	13	2.2485	0.0514
18	14	1.0623	0.0387
17	15	1.1993	0.0038
10	16	0.8372	0.0218
9	17	0.0402	1.0697

** $p < .01$

TABLE 12

RESULTS OF DISCRIMINANT ANALYSIS OF TRIAL 4 DATA

Predictor Variable	Rank	Independent *F*	*F* to Remove
12	1	12.1135**	12.1135**
14	2	0.1864	7.5862
13	3	0.9128	3.9102
9	4	0.0352	1.5959
6	5	0.6380	0.7626
4	6	0.1601	0.8658
5	7	0.3509	3.9986
7	8	0.0336	5.5627
17	9	0.0434	1.8671
16	10	2.1188	0.9093
15	11	0.2036	0.2991
2	12	0.2403	0.0222
10	13	0.8333	0.0283
11	14	0.9670	0.0118
1	15	0.1740	0.0028

* $p < .05$

** $p < .01$

TABLE 13

RESULTS OF DISCRIMINANT ANALYSIS OF TRIAL 5 DATA

Predictor Variable	Rank	Independent *F*	*F* to Remove
17	1	1.9544	1.9544
13	2	1.0245	2.2557
16	3	0.0453	0.6628
1	4	0.4046	0.8194
14	5	0.0804	0.1632
9	6	0.2925	0.3906
12	7	0.0021	0.1773
3	8	0.2769	0.0940
5	9	1.5607	0.1231
18	10	0.8180	0.0862
11	11	0.0070	0.0481
8	12	0.1680	0.0296
4	13	0.3940	0.0180
7	14	0.2837	0.1508
6	15	0.0226	0.0223

TABLE 14

RESULTS OF DISCRIMINANT ANALYSIS OF TRIAL 6 DATA

Predictor Variable	Rank	Independent F	F to Remove
12	1	56.8298**	56.8298**
9	2	0.4641	10.1199**
5	3	2.8173	7.2512**
15	4	0.3019	3.0168
7	5	0.1354	2.1874
6	6	1.2977	2.3116
1	7	0.0092	0.9782
13	8	0.4876	0.4713
14	9	7.6743**	0.3681
11	10	12.9722**	1.8159
18	11	0.7796	0.2774
17	12	0.4290	0.9224
4	13	0.6937	0.2195
16	14	2.2763	0.0360
3	15	0.2066	0.0347
8	16	0.8717	0.0147

* $p < .01$

TABLE 15

PERCENTAGE CORRECT CLASSIFICATION BY DISCRIMINANT FUNCTION ON EACH TRIAL

Trial	Percentage Correct Classification			
	After First Variable Enters Function		After Last Variable Enters Function	
	Recalled	Not Recalled	Recalled	Not Recalled
Trial 1	73.6	65.2	71.7	75.6
Trial 2	65.8	69.6	67.1	82.6
Trial 3	56.7	76.4	67.6	76.5
Trial 4	89.1	68.4	93.6	55.6
Trial 5	78.9	50.0	74.3	60.0
Trial 6	93.1	60.0	99.1	60.0

trial, in addition to this variable, the abstractness of the response concept and the maximum length of paths to primitives from the response concept. It should be noted that by the fourth trial recall was better than 90% and thus a given _S_ was on the average only incorrectly recalling about two response concepts per trial.

Consistency of Epistemological Structures

Each _S_ defined the set of 50 set theoretic concepts on two occasions. The first occasion was before the start of the sequence of experiments and the second occasion was about one month later following the conclusion of the final experiment. In an effort to indicate the consistency, across time, of the _S_'s epistemological structurings of the set of concepts, the percentage of agreement in defining the set of concepts on the two occasions was computed for each _S_. The method of computation of this measure of consistency as well as two other measures of consistency is given in Appendix F. The mean percentage of agreement for the _S_s was 89.68 with standard deviation of 4.23. Two different measures of consistency of definition across _S_s were computed. The first of these measures was the mean percentage of agreement across _S_s. For the initial defining this mean was 27.57 with standard deviation of 2.76 and for the final defining the mean was 26.38 with standard deviation of 4.87. As can be observed from an examination of the methods of computing the above two means in Appendix H, it is possible for the

percentage of agreement in the defining of a concept to be zero if one S's definition of the concept does not agree at all with any other S's definition of the concept even though it could be the case that there was perfect agreement on the definition of the concept among the other Ss. To avoid this drawback another measure of consistency of definition between Ss, as shown in Appendix H, was computed. This measure was the mean of the mean percentage of agreement in defining the set of concepts between every possible pair of Ss. This mean was 62.07 with standard deviation of 3.10 for the first occasion and 60.37 with standard deviation of 4.26 for the second occasion.

CHAPTER 3

GENERAL DISCUSSION AND A PROPOSAL

This study has been based on a rather limited graph theoretical model of epistemological structure: that imposed upon a set of concepts by their definitions. A computer program, discussed earlier, made possible the construction of each S's epistemological structuring of 50 concepts from set theory and the identification of many associated structural characteristics. A number of these graphical characteristics -- the distance between pairs of concepts, the number of paths between concept pairs, and the abstractness of the concepts -- were manipulated in order to assess the effects of these characteristics on recall. An attempt was also made to indicate some possible effects of the nonmanipulated graphical characteristics.

The reasoning behind the experiments was the idea that if the model accurately represents some aspects of the epistemological information a S uses in memory tasks, then the quantitative identifications made possible by the model would allow one to manipulate variables which affect a S's performance in such memory tasks. The suitability of the model is rather straightforward when variations in graphical characteristics, by appropriate selection of stimulus

materials, result in concomitant variations in performance but rather difficult to evaluate when there are no variations in performance. In this latter case, it is a moot point whether such characteristics are invalid identifications or, while valid identifications, are not of differential importance to performance.

In this chapter, the results of the series of experiments reported in the previous chapter will be discussed from the perspective of the suitability of the model of epistemological structure on which these experiments were based. In addition, a more general graph theoretical model of epistemological structure is proposed.

The Model and the Results

The major result of the studies reported in this work was that semantic distance as quantified by mean path length between concepts in the graphical representation of a S's epistemological structure was a potent variable affecting recall of concepts in a paired associate task. A corollary of this result was that recall for response terms of concept pairs connected by paths was decisively better than those of concept pairs not connected by paths. Both of these results lend weight to the accuracy of the model of epistemological structure presented here.

As mentioned earlier, the negative findings are much more difficult to interpret. For example, it is usually

possible to postulate a recall strategy based on the model which could account for any particular negative finding. Such postulations are, of course, only conjectures and should be subjected to further experimental analysis.

The positive findings of these studies not only provide evidence in support of the particular model of epistemological structure presented here but also validate the distinctive approach of these investigations. This approach was based on the assumption that each S's particular structuring of the set of concepts was of primary importance to his subsequent performance in memory tasks involving these concepts. Note as supportive evidence for this approach the high level of agreement of definitions within Ss but the comparatively low level of agreement between Ss. This approach contrasts greatly, as was discussed in the first chapter, with that of other researchers in the area of semantic memory. It was adopted because of the greater experimental control it made possible and the thought that lack of such control might well have led to the present controversies surrounding the reaction time studies discussed earlier. In any case, when dealing with a phenomenon as complex and intractable as human memory, any additional control over the experimental situation is of tremendous importance.

The major benefit of this model of epistemological structure was that it allowed the quantification of numerous structural characteristics by virtue of its graphical

formulation. Such quantifications made possible the experimental investigations which comprise this work. There are numerous other things which a model of epistemological structure makes possible. It allows one to formulate in explicit terms recall strategies which a S might use in retrieving information from semantic memory, e.g., Quillian's (1969) breadth-first search. It allows the identification of the many logical dependencies which are contained within the epistemological structure. These dependencies, while important in their own right, might well be important to cognitive activities like memory but such importance is unlikely to be identified without models of this structure and computerized techniques for accessing structural characteristics. In addition, a graphical model allows the use of the extensive literature of graph theory in identifying the structural characteristics. Finally, and possibly most importantly, to the extent the model is an accurate representation of a S's epistemological structure, it allows an extensive analysis of the structural characteristics of a S's epistemological structure without requiring the continual presence of the S, freeing the investigator of many restrictions usually associated with the use of human Ss, and thus might well make possible much better controlled and more informative subsequent experiments.

Based on the success of the model used in these studies and the many benefits of such models, as discussed above, a more complex graph theoretical model of

epistemological structure is proposed in the next section. This model will allow the identification of a host of relationships which logically follow from the definitions of a set of concepts, from the explicit formulation of the nondefining characteristics of the concepts' referents, and from the use of the concepts in theorems and proofs of theorems.

A Proposal

In this section, an extension of the graph theoretical model of epistemological structure used in this study is proposed. It will make possible the representation of epistemological relationships between the genera of concepts, the differentiae of concepts, the nondefining characteristics of concepts, theorems, and proofs of theorems. It is hoped that the proposed model will facilitate the quantitative identification of the many epistemological relationships inherent in an individual's knowledge of an area of mathematics and lead to an understanding of the importance of such relationships to cognitive activities like memory.

Like the previous graphical model of epistemological structure, this model is composed of points and directed edges between the points, i.e., it is a directed graph, but, unlike the previous model, in addition to points which are associated with concepts there are points which are associated with theorems and points which are associated with proofs.

These three types of points will be termed concept points, theorem points, and proof points. Since more than one type of relationship can exist between the points, the directed edges between points are labeled. A concept point can have directed edges to the concept points of the genus of its definition, to the concept points of the differentia of its definition, and to the concept points which comprise its nondefining characteristics. A theorem point can have directed edges to concept points of the concepts used in the statement of the theorem and to proof points associated with proofs of the theorem. A proof point can have directed edges to concept points of the concepts used in the proof and to theorem points of theorems cited in the proof. Thus, while there are three types of points, there are seven different types of labeled edges. Table 16 depicts the symbols which are used to represent the points and directed edges in the example graph segment shown in Figure 13.

One must note the wealth of epistemological relationships which can be identified by use of the proposed model. Not only can one ascertain definitional dependencies between concepts as with the previous model but these dependencies can be further classified as those involving genus links, differentia links, or both; also included are dependencies resulting from links due to nondefining characteristics of the concepts. Furthermore, with the inclusion of theorems and proofs in the model, the types of relationships become myriad. Consider, for example, that the dependencies of

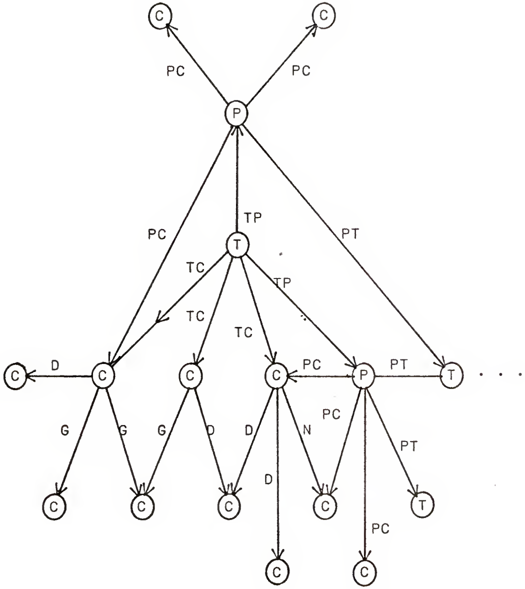


Figure 13. Graph Segment Demonstrating Types of Relationships in Proposed Model of Epistemological Structure

TABLE 16

SYMBOLS USED TO REPRESENT POINTS AND DIRECTED EDGES

Points	Symbol
Concept	C
Theorem	T
Proof	P

Directed Edges	Symbol
From a concept point, C_i, to a concept point, C_j, of the genus of C_i.	G
From a concept point, C_i, to a concept point, C_j, of the differentia of C_i.	D
From a concept point, C_i, to a concept point, C_j, of the nondefining characteristics of C_i.	N
From a theorem point, T_i, to a concept point, C_j, used in the statement of T_i.	TC
From a theorem point, T_i, to a proof point, P_j, employed as a proof of T_i.	TP
From a proof point, P_i, to a concept point, C_j, used in the proof P_i.	PC
From a proof point, P_i, to a theorem point, T_j, used in the proof P_i.	PT

theorems on proofs, other theorems, and concepts can be identified as well as the dependencies of proofs upon other proofs, theorems and concepts. In addition to these dependencies, the model makes possible the identification of the many graphical characteristics of the model. The primary purpose of this model of epistemological structure is the same as that of the previous model: to aid the experimental investigation of the importance of epistemological relationships in complex cognitive activities like those involved in human memory by formulating these relationships in explicit mathematical terms. The model is presented here as a means of expressing a direction for the continuation of the research reported in this work.

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APPENDICES

APPENDIX A

A PORTION OF THE COMPUTER LISTING OF PATHS TO PRIMITIVE CONCEPTS FOR THE GRAPH THEORY DATA AND A GRAPH OF THE ASSOCIATED EPISTEMOLOGICAL STRUCTURE TO DEMONSTRATE ITS COMPLEXITY

```
ADJACENCY
    ISOMORPHIC
        SELF-COMPLEMENTARY
        INVARIANT
ADJACENT
    INTERSECTION GRAPH
        CLIQUE GRAPH
        INTERSECTION NUMBER
    COMPLETE
        CLIQUE
            CLIQUE GRAPH
    COMPLEMENT
        SELF-COMPLEMENTARY
    SQUARE
DISTINCT
    CYCLE
        ACYCLIC
            TREE
                CENTROID
                CENTROID POINT
                    CENTROID
                WEIGHT
                    CENTROID POINT
                        CENTROID
                BRANCH
                    WEIGHT
                        CENTROID POINT
                            CENTROID
        CIRCUMFERENCE
        GIRTH
    PATH
        GEODESIC
            DIAMETER
        DISTANCE D(U,V)
            ECCENTRICITY
                CENTRAL POINT
                    CENTER
                RADIUS
                    CENTRAL POINT
                        CENTER
            SQUARE
        CONNECTED
            ECCENTRICITY
                CENTRAL POINT
                    CENTER
                RADIUS
                    CENTRAL POINT
                        CENTER
            TREE
                CENTROID
                CENTROID POINT
                    CENTROID
```

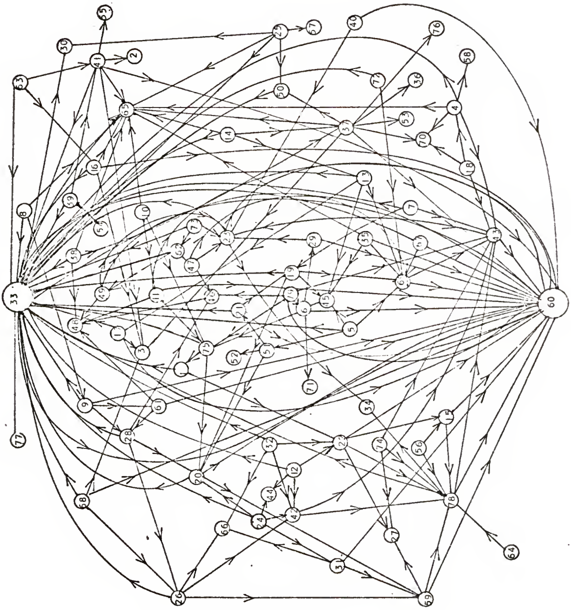
```
        WEIGHT
          CENTROID POINT
            CENTROID
        BRANCH
          WEIGHT
            CENTROID POINT
              CENTROID
      NONSEPARABLE
        BLOCK
      DIAMETER
      COMPONENT
        BRIDGE
        CUTPOINT
          NONSEPARABLE
            BLOCK
  TRAIL
FINITE
  DIGRAPH
  GRAPH
    CENTER
    CENTRAL POINT
      CENTER
    ECCENTRICITY
      CENTRAL POINT
        CENTER
      RADIUS
        CENTRAL POINT
          CENTER
    TREE
      CENTROID
      CENTROID POINT
        CENTROID
      WEIGHT
        CENTROID POINT
          CENTROID
      BRANCH
        WEIGHT
          CENTROID POINT
            CENTROID
    ACYCLIC
      TREE
        CENTROID
        CENTROID POINT
          CENTROID
        WEIGHT
          CENTROID POINT
            CENTROID
        BRANCH
          WEIGHT
            CENTROID POINT
              CENTROID
    BLOCK
```

```
BRIDGE
CUTPOINT
   NONSEPARABLE
      BLOCK
CLIQUE GRAPH
INTERSECTION NUMBER
CLIQUE
   CLIQUE GRAPH
BIGRAPH
   COMPLETE BIGRAPH
TOTALLY DISCONNECTED
COMPLETE
   CLIQUE
      CLIQUE GRAPH
SELF-COMPLEMENTARY
COMPLEMENT
   SELF-COMPLEMENTARY
REGULAR
   TOTALLY DISCONNECTED
DEGREE
   TOTALLY DISCONNECTED
   ENDPOINT
      BRANCH
         WEIGHT
            CENTROID POINT
               CENTROID
   ISOLATED
   MAXIMUM DEGREE
      REGULAR
         TOTALLY DISCONNECTED
   MINIMUM DEGREE
      REGULAR
         TOTALLY DISCONNECTED
SQUARE
DIAMETER
DISTANCE D(U,V)
   ECCENTRICITY
      CENTRAL POINT
         CENTER
      RADIUS
         CENTRAL POINT
            CENTER
   SQUARE
CIRCUMFERENCE
GIRTH
COMPONENT
   BRIDGE
   CUTPOINT
      NONSEPARABLE
         BLOCK
CONNECTED
   ECCENTRICITY
```

```
        CENTRAL POINT
          CENTER
        RADIUS
          CENTRAL POINT
            CENTER
      TREE
        CENTROID
        CENTROID POINT
          CENTROID
        WEIGHT
          CENTROID POINT
            CENTROID
        BRANCH
          WEIGHT
            CENTROID POINT
              CENTROID
      NONSEPARABLE
        BLOCK
      DIAMETER
      COMPONENT
        BRIDGE
        CUTPOINT
          NONSEPARABLE
            BLOCK
    WALK
      CYCLE
        ACYCLIC
          TREE
            CENTROID
            CENTROID POINT
              CENTROID
            WEIGHT
              CENTROID POINT
                CENTROID
            BRANCH
              WEIGHT
                CENTROID POINT
                  CENTROID
        CIRCUMFERENCE
        GIRTH
      LENGTH
        DIAMETER
        DISTANCE D(U,V)
          ECCENTRICITY
            CENTRAL POINT
              CENTER
            RADIUS
              CENTRAL POINT
                CENTER
          SQUARE
        CIRCUMFERENCE
        GIRTH
```

```
PATH
   GEODESIC
      DIAMETER
   DISTANCE D(U,V)
      ECCENTRICITY
         CENTRAL POINT
            CENTER
         RADIUS
            CENTRAL POINT
               CENTER
      SQUARE
   CONNECTED
      ECCENTRICITY
         CENTRAL POINT
            CENTER
         RADIUS
            CENTRAL POINT
               CENTER
      TREE
         CENTROID
         CENTROID POINT
            CENTROID
         WEIGHT
            CENTROID POINT
               CENTROID
         BRANCH
            WEIGHT
               CENTROID POINT
                  CENTROID
      NONSEPARABLE
         BLOCK
      DIAMETER
      COMPONENT
         BRIDGE
         CUTPOINT
            NONSEPARABLE
               BLOCK
TRAIL
OPEN
CLOSED
   CYCLE
      ACYCLIC
         TREE
            CENTROID
            CENTROID POINT
               CENTROID
            WEIGHT
               CENTROID POINT
                  CENTROID
            BRANCH
               WEIGHT
                  CENTROID POINT
```

```
                                        CENTROID
                CIRCUMFERENCE
                GIRTH
            OPEN
        SPANNING GRAPH
        SUPERGRAPH
        SUBGRAPH
            BLOCK
            CLIQUE
                CLIQUE GRAPH
            COMPONENT
                BRIDGE
                CUTPOINT
                    NONSEPARABLE
                        BLOCK
            INDUCED SUBGRAPH
            SPANNING GRAPH
            SUPERGRAPH
        INVARIANT
        ISOMORPHIC
            SELF-COMPLEMENTARY
            INVARIANT
        DIGRAPH
INCIDENT
    DEGREE
        TOTALLY DISCONNECTED
        ENDPOINT
            BRANCH
                WEIGHT
                    CENTROID POINT
                        CENTROID
        ISOLATED
        MAXIMUM DEGREE
            REGULAR
                TOTALLY DISCONNECTED
        MINIMUM DEGREE
            REGULAR
                TOTALLY DISCONNECTED
    WALK
        CYCLE
            ACYCLIC
                TREE
                    CENTROID
                    CENTROID POINT
                        CENTROID
                    WEIGHT
                        CENTROID POINT
                            CENTROID
                    BRANCH
                        WEIGHT
                            CENTROID POINT
                                CENTROID
```



DIGRAPH OF EPISTEMOLOGICAL STRUCTURE
OF GRAPH THEORY CONCEPTS

KEY

1. acyclic
2. adjacency
3. adjacent
4. bigraph
5. block
6. branch
7. bridge
8. center
9. central point
10. centroid

11. centroid point
12. circumference
13. clique
14. clique graph
15. closed
16. complete
17. complete graph
18. complement
19. component
20. connected

21. cutpoint
22. cycle
23. degree
24. diameter
25. digraph
26. distance
27. distinct
28. eccentricity
29. endpoint
30. finite

31. geodesic
32. girth
33. graph
34. incident
35. induced subgraph
36. intersection
37. intersection graph
38. intersection number
39. invariant
40. isolated

41. isomorphic
42. length
43. line
44. longest
45. maximal
46. maximum
47. maximum degree
48. minimum
49. minimum degree
50. nonempty

51. nonseparable
52. nontrivial
53. null set
54. number
55. one-to-one correspondence
56. open
57. ordered
58. partition
59. path
60. point

61. radius
62. regular
63. self-complementary
64. sequence
65. set
66. shortest
67. spanning graph
68. square
69. subgraph
70. subset

71. subtree
72. supergraph
73. totally disconnected
74. trail
75. tree
76. union
77. unordered
78. walk
79. weight

APPENDIX B

A PORTION OF THE COMPUTER OUTPUT FROM A RUN WITH THE GRAPH THEORY DATA

```
  1
ELT_NAME =                 ACYCLIC
ELT'S PRESENT FATHERS:
GRAPH
CYCLE
OUT_DEGREE:     2
ELT'S SONS:
TREE
IN_DEGREE:     1
ELT DATA
A GRAPH IS ACYCLIC IF IT HAS NO CYCLES.

 ( 27,  2) ( 30,  2) ( 50,  2) ( 60,  2) ( 60,  2) ( 65,  2)
 ( 77,  2) ( 34,  3) ( 64,  3) ( 60,  3) ( 60,  3) ( 60,  4)
 ( 34,  4) ( 60,  4) ( 30,  4) ( 64,  4) ( 50,  4) ( 65,  4)
 ( 60,  4) ( 77,  4) ( 30,  5) ( 60,  5) ( 65,  5) ( 60,  5)
 ( 50,  5) ( 77,  5)
MEAN PATH LENGTH:    3.538

  2
ELT_NAME =                 ADJACENCY
ELT'S PRESENT FATHERS:
NONE PRESENT.
OUT_DEGREE:     0
ELT'S SONS:
ISOMORPHIC
IN_DEGREE:     1
ELT DATA
PRIMITIVE


  3
ELT_NAME =                 ADJACENT
ELT'S PRESENT FATHERS:
NONE PRESENT.
OUT_DEGREE:     0
ELT'S SONS:
INTERSECTION GRAPH
COMPLETE
COMPLEMENT
SQUARE
IN_DEGREE:     4
ELT DATA
PRIMITIVE


  4
ELT_NAME =                 BIGRAPH
ELT'S PRESENT FATHERS:
GRAPH
POINT
```

```
SET
PARTITION
SUBSET
LINE
OUT_DEGREE:    6
ELT'S SONS:
COMPLETE BIGRAPH
IN_DEGREE:    1
ELT DATA
A BIGRAPH IS A GRAPH WHOSE POINT SET V CAN BE PARTITIONED IN
TO TWO SUBSETS V1 AND V2 S.T. EVERY LINE OF G JOINS V1 WITH
V2.
 ( 58.  1) ( 60.  1) ( 65.  1) ( 70.  1) ( 60.  2) ( 60.  2)
 ( 30.  2) ( 65.  2) ( 50.  2) ( 77.  2)
MEAN PATH LENGTH:   1.600

  5
ELT_NAME =                BLOCK
ELT'S PRESENT FATHERS:
GRAPH
MAXIMAL
NONSEPARABLE
SUBGRAPH
OUT_DEGREE:    4
ELT'S SONS:
NONE PRESENT.
IN_DEGREE:    0
ELT DATA
A BLOCK OF A GRAPH IS A MAXIMAL NONSEPARABLE SUBGRAPH.

 ( 45.  1) ( 30.  2) ( 50.  2) ( 52.  2) ( 60.  2) ( 60.  2)
 ( 65.  2) ( 77.  2) ( 50.  3) ( 60.  3) ( 30.  3) ( 60.  3)
 ( 60.  3) ( 65.  3) ( 60.  3) ( 77.  3) ( 50.  4) ( 27.  4)
 ( 60.  4) ( 60.  4) ( 30.  4) ( 45.  4) ( 65.  4) ( 65.  4)
 ( 30.  4) ( 60.  4) ( 77.  4) ( 77.  4) ( 50.  4) ( 60.  5)
 ( 34.  5) ( 65.  5) ( 60.  5) ( 50.  5) ( 60.  5) ( 60.  5)
 ( 77.  5) ( 30.  5) ( 64.  5) ( 30.  6) ( 60.  6) ( 50.  6)
 ( 30.  6) ( 60.  6) ( 30.  6) ( 60.  6) ( 65.  6) ( 65.  6)
 ( 27.  6) ( 65.  6) ( 50.  6) ( 50.  6) ( 60.  6) ( 60.  6)
 ( 77.  6) ( 77.  6) ( 60.  6) ( 77.  6) ( 60.  7) ( 64.  7)
 ( 34.  7) ( 30.  8) ( 60.  8) ( 60.  8) ( 77.  8) ( 50.  8)
 ( 65.  8)
MEAN PATH LENGTH:   4.835

  6
ELT_NAME =                BRANCH
ELT'S PRESENT FATHERS:
TREE
MAXIMAL
SUBTREE
ENDPOINT
POINT
OUT_DEGREE:    5
```

```
ELT'S SONS:
WEIGHT
IN_DEGREE:     1
ELT DATA
A BRANCH AT A POINT U OF A TREE T IS A MAXIMAL SUBTREE CONTA
INING U AS AN END-  POINT.

 ( 45,  1) ( 60,  1) ( 71,  1) ( 60,  2) ( 34,  3) ( 30,  3)
 ( 60,  3) ( 60,  3) ( 60,  3) ( 65,  3) ( 50,  3) ( 77,  3)
 ( 30,  4) ( 30,  4) ( 27,  4) ( 60,  4) ( 27,  4) ( 60,  4)
 ( 60,  4) ( 30,  4) ( 60,  4) ( 60,  4) ( 60,  4) ( 50,  4)
 ( 65,  4) ( 65,  4) ( 65,  4) ( 50,  4) ( 50,  4) ( 77,  4)
 ( 77,  4) ( 77,  4) ( 60,  5) ( 60,  5) ( 60,  5) ( 64,  5)
 ( 64,  5) ( 34,  5) ( 34,  5) ( 50,  6) ( 64,  6) ( 60,  6)
 ( 60,  6) ( 34,  6) ( 30,  6) ( 65,  6) ( 65,  6) ( 30,  6)
 ( 50,  6) ( 60,  6) ( 60,  6) ( 60,  6) ( 77,  6) ( 77,  6)
 ( 60,  7) ( 50,  7) ( 65,  7) ( 30,  7) ( 60,  7) ( 77,  7)
MEAN PATH LENGTH:   4.600

  7
ELT_NAME =                 BRIDGE
ELT'S PRESENT FATHERS:
GRAPH
LINE
COMPONENT
OUT_DEGREE:     3
ELT'S SONS:
NONE PRESENT.
IN_DEGREE:     0
ELT DATA
A BRIDGE OF A GRAPH IS A LINE WHOSE REMOVAL INCREASES THE NU
MBER OF COMPONENTS.
 ( 30,  2) ( 45,  2) ( 50,  2) ( 60,  2) ( 60,  2) ( 65,  2)
 ( 77,  2) ( 30,  3) ( 50,  3) ( 60,  3) ( 60,  3) ( 65,  3)
 ( 60,  3) ( 77,  3) ( 60,  4) ( 50,  4) ( 50,  4) ( 60,  4)
 ( 60,  4) ( 27,  4) ( 60,  4) ( 65,  4) ( 65,  4) ( 30,  4)
 ( 30,  4) ( 77,  4) ( 77,  4) ( 64,  5) ( 60,  5) ( 34,  5)
 ( 60,  6) ( 30,  6) ( 60,  6) ( 50,  6) ( 77,  6) ( 65,  6)
MEAN PATH LENGTH:   3.833

  8
ELT_NAME =                 CENTER
ELT'S PRESENT FATHERS:
GRAPH
SET
CENTRAL POINT
OUT_DEGREE:     3
ELT'S SONS:
NONE PRESENT.
IN_DEGREE:     0
ELT DATA
THE CENTER OF A GRAPH G IS THE SET OF ALL CENTRAL POINTS.
```

```
 ( 65,  1) ( 50,  2) ( 60,  2) ( 60,  2) ( 30,  2) ( 65,  2)
 ( 77,  2) ( 60,  3) ( 60,  3) ( 48,  3) ( 60,  3) ( 30,  3)
 ( 50,  3) ( 65,  3) ( 46,  3) ( 77,  3) ( 30,  4) ( 60,  4)
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 ( 46,  4) ( 77,  4) ( 60,  5) ( 50,  5) ( 50,  5) ( 27,  5)
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 ( 60,  8) ( 77,  8) ( 65,  8) ( 77,  8)
MEAN PATH LENGTH:    5.711

  9
ELT_NAME =                 CENTRAL POINT
ELT'S PRESENT FATHERS:
POINT
ECCENTRICITY
RADIUS
GRAPH
OUT_DEGREE:     4
ELT'S SONS:
CENTER
IN_DEGREE:     1
ELT DATA
A POINT V IS A CENTRAL POINT IF THE ECCENTRICITY OF THE POIN
T IS EQUAL TO THE   RADIUS OF THE GRAPH.

 ( 60,  1) ( 46,  2) ( 48,  2) ( 50,  2) ( 30,  2) ( 60,  2)
 ( 60,  2) ( 60,  2) ( 65,  2) ( 77,  2) ( 60,  3) ( 60,  3)
 ( 60,  3) ( 30,  3) ( 60,  3) ( 46,  3) ( 65,  3) ( 66,  3)
 ( 50,  3) ( 77,  3) ( 60,  4) ( 50,  4) ( 60,  4) ( 60,  4)
 ( 60,  4) ( 50,  4) ( 50,  4) ( 60,  4) ( 60,  4) ( 60,  4)
 ( 27,  4) ( 27,  4) ( 65,  4) ( 65,  4) ( 65,  4) ( 30,  4)
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 ( 34,  5) ( 34,  5) ( 60,  5) ( 34,  5) ( 60,  5) ( 64,  5)
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 ( 30,  6) ( 64,  6) ( 60,  6) ( 64,  6) ( 34,  6) ( 64,  6)
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 ( 65,  7) ( 50,  7) ( 60,  7) ( 30,  7) ( 60,  7) ( 60,  7)
 ( 60,  7) ( 50,  7) ( 60,  7) ( 30,  7) ( 50,  7) ( 77,  7)
 ( 65,  7) ( 77,  7) ( 30,  7) ( 77,  7)
MEAN PATH LENGTH:   4.919

 10
ELT_NAME =               CENTROID
ELT'S PRESENT FATHERS:
TREE
SET
CENTROID POINT
OUT_DEGREE:     3
ELT'S SONS:
NONE PRESENT.
IN_DEGREE:     0
ELT DATA
THE CENTROID OF A TREE T IS THE SET OF ALL CENTROID POINTS O
F T.
 ( 65,  1) ( 60,  2) ( 48,  2) ( 50,  3) ( 46,  3) ( 60,  3)
 ( 60,  3) ( 60,  3) ( 30,  3) ( 65,  3) ( 77,  3) ( 50,  4)
 ( 50,  4) ( 30,  4) ( 30,  4) ( 60,  4) ( 60,  4) ( 45,  4)
 ( 60,  4) ( 60,  4) ( 60,  4) ( 27,  4) ( 60,  4) ( 60,  4)
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 ( 71,  4) ( 50,  4) ( 77,  4) ( 77,  4) ( 77,  4) ( 60,  5)
 ( 60,  5) ( 50,  5) ( 34,  5) ( 50,  5) ( 60,  5) ( 30,  5)
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 ( 60,  5) ( 27,  5) ( 65,  5) ( 60,  5) ( 65,  5) ( 30,  5)
 ( 65,  5) ( 50,  5) ( 60,  5) ( 34,  5) ( 77,  5) ( 60,  5)
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 ( 30,  8) ( 60,  8) ( 65,  8) ( 64,  8) ( 65,  8) ( 34,  8)
 ( 50,  8) ( 64,  8) ( 60,  8) ( 30,  8) ( 60,  8) ( 60,  8)
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 ( 60,  8) ( 60,  8) ( 50,  8) ( 77,  8) ( 34,  8) ( 64,  8)
 ( 77,  8) ( 60,  8) ( 77,  8) ( 60,  9) ( 60,  9) ( 60,  9)
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 ( 50,  9) ( 65,  9) ( 34,  9) ( 30,  9) ( 77,  9) ( 65,  9)
 ( 60,  9) ( 30,  9) ( 77,  9) ( 60,  9) ( 50,  9) ( 77,  9)
 ( 50, 10) ( 60, 10) ( 60, 10) ( 30, 10) ( 65, 10) ( 77, 10)
```

APPENDIX C

THE SET THEORETIC CONCEPTS DEFINED BY EACH SUBJECT

ANTISYMMETRIC

ASSOCIATIVE

BINARY OPERATION

CARTESIAN PRODUCT

COMMUTATIVE

COMPARABLE

COMPLEMENT

COMPOSITION

CONTAINS

DIFFERENCE

DISJOINT

DISTRIBUTIVE

DOMAIN

EQUALITY

EQUIVALENCE CLASS

EQUIVALENCE RELATION

FUNCTION

GREATEST LOWER BOUND

INDEX SET

INTERSECTION

INTO

INVERSE

ELEMENT

LEAST UPPER BOUND

LOWER BOUND

MAXIMAL ELEMENT

MINIMAL ELEMENT

NULL SET

ONE-TO-ONE

ONTO

OPERATION

ORDERED PAIR

PARTIAL ORDER

PARTITION

POWER SET

PREDECESSOR

PROPER SUBSET

QUASI ORDER

RANGE

REFLEXIVE

RELATION

SET

SINGLETON

SUBSET

SUCCESSOR

SYMMETRIC

TOTAL ORDER

TRANSITIVE

UNION

UPPER BOUND

WELL ORDERED

APPENDIX D

SAMPLES OF LISTS USED IN THE EXPERIMENTS

SAMPLE LIST EXPERIMENT I

MAXIMAL ELEMENT - UNION

EQUIVALENCE CLASS - CARTESIAN PRODUCT

COMPOSITION - DISJOINT

RANGE - INDEX SET

POWER SET - SUBSET

SUCCESSOR - SET

ANTISYMMETRIC - RELATION

PARTIAL ORDER - REFLEXIVE

WELL ORDERED - ONE-TO-ONE

PREDECESSOR - ELEMENT

DISJOINT - NULL SET

DIFFERENCE - INTO

ASSOCIATIVE - BINARY OPERATION

UPPER BOUND - FUNCTION

EQUALITY - COMPLEMENT

QUASI ORDER - INVERSE

PARTITION - DOMAIN

GREATEST LOWER BOUND - CONTAINS

COMMUTATIVE - SINGLETON

ONTO - PROPER SUBSET

SAMPLE LIST EXPERIMENT 2

WELL ORDERED - RELATION

MINIMAL ELEMENT - OPERATION

INVERSE - CARTESIAN PRODUCT

PARTITION - DOMAIN

DISJOINT - RANGE

FUNCTION - SET

INTO - SUBSET

INDEX SET - LOWER BOUND

GREATEST LOWER BOUND - TRANSITIVE

COMMUTATIVE - SINGLETON

ASSOCIATIVE - BINARY OPERATION

QUASI ORDER - ONTO

LEAST UPPER BOUND - NULL SET

EQUALITY - COMPLEMENT

EQUIVALENCE RELATION - ELEMENT

COMPOSITION - ORDERED PAIR

INTERSECTION - SYMMETRIC

CONTAINS - POWER SET

TOTAL ORDER - MAXIMAL ELEMENT

ONE-TO-ONE - UNION

SAMPLE LIST EXPERIMENT 3

DISJOINT

SUBSET

BINARY OPERATION

SET

MINIMAL ELEMENT

ASSOCIATIVE

LEAST UPPER BOUND

WELL ORDERED

SUCCESSOR

UNION

DIFFERENCE

PREDECESSOR

INTERSECTION

ELEMENT

NULL SET

CONTAINS

COMMUTATIVE

TOTAL ORDER

EQUIVALENCE CLASS

ORDERED PAIR

APPENDIX E

THE DECODING TASK

A B C D E F G H I J K L M N O P Q R S T U V W X Y Z .
* ; ⊃ U ↓ ∩ α ⊂ O ÷ ω ↑ ∘ Γ ? L _ T ρ → ~ □ (\ ι ∧ '

∩→↓T ⊂Oρ U↓→⊂ ∩T?∘ α?~→ ?Γ ρ↓L→↓∘;↓T 2 1764 OΓ →⊂↓ ρO\→ι
∩OTρ→ ι↓*T ?∩ ⊂Oρ *α↓ O→ (*ρ ∩?~ΓU →⊂*→ ⊂*∘O↑→?Γ ⊂*U ↑↓∩→
;↓⊂OΓU * ∘*ρρ ?∩ L*L↓Tρ OΓ OΓU↓ρ⊃TO;↓U ⊃?Γ∩~ρO?Γ *ΓU *;?~→
ρO\→ι ⊂~α↓ ∘*Γ~ρ⊃TOL→ ;??ωρ ∩~↑↑ ?∩ ∘*→⊂↓∘*→O⊃ρ' *Γ *U↓_~*→↓
↓UO→O?Γ ?∩ ⊂Oρ (?Tωρ Oρ Γ?(OΓ LT?αT↓ρρ' →⊂↓ ρ→*→↓ ?∩ ⊂Oρ
L*L↓Tρ →↓ρ→O∩O↓ρ →? →⊂↓ U?∘↓ρ→O⊃ UO∩∩O⊃~↑→O↓ρ ~ΓU↓T (⊂O⊃⊂
→⊂↓ ↑*ρ→ →⊂OTU ?∩ ⊂Oρ ↑O∩↓ ⊂*U ;↓↓Γ ↑O□↓U OΓΓ~∘↓T*;↑↓ UOΓΓ↓T
L↑*→↓ρ (O→⊂ →⊂↓ T↓∘*OΓρ ?∩ U↓ρO⊃⊃*→↓U ~Γ□O?↑*→↓U ⊃⊂?Lρ (↓T↓
∩?~ΓU ;~TO↓U OΓ →⊂↓ ∘?~Γ→*OΓ?~ρ LO↑↓ρ ?∩ L*L↓Tρ *ΓU UOρ⊂↓ρ
↓Γ?~α⊂ →? ρ~LL↑ι * ↑*Tα↓ ⊂?~ρ↓⊂?↑U (↓T↓ U~α ?~→ ∩T?∘ →⊂↓
⊃?Γ∩~ρO?Γ' U~TOΓα ⊂Oρ ↑*ρ→ L↓TO?U ⊂*∘O↑→?Γ ↑O□↓U *ρ * T↓⊃↑~ρ↓
OαΓ?TOΓα →⊂↓ ∘↓*↑ρ ρ⊂?□↓U *→ ⊂O∘ *ρ ⊂↓ (?Tω↓U ?;ρ↓ρρ↓U ;ι
→⊂↓ UT↓*∘ →⊂*→ →⊂↓ ↑*ρ→ →T↓∘↓ΓU?~ρ ↓∩∩?T→ ?∩ ⊂Oρ ∘*αΓO∩O⊃↓Γ→
α↓ΓO~ρ (?~↑U O∘∘?T→*↑O∧↓ ;?→⊂ ⊂O∘ρ↓↑∩ *ΓU ⊂Oρ ;↓↑?□↓U OT↓↑*ΓU
*ΓU ρ→*ΓU ∩?T↓□↓T ~Γρ⊂*ω↓Γ *ρ →⊂↓ αT↓*→↓ρ→ ∘*→⊂↓∘*→O⊃*↑
⊃?Γ→TO;~→O?Γ →? ρ⊃O↓Γ⊃↓ ρOΓ⊃↓ →⊂↓ LTOΓ⊃OLO* ?∩ Γ↓(→?Γ'

A B C D E F G H I J K L M N O P Q R S T U V W X Y Z .
* ; ⊃ ∪ ↓ ∩ ⍺ ⊂ ○ ÷ ⍵ ↑ ∘ ⌈ ? ⌊ _ ⊤ ⍴ → ~ ⎕ (\ ⍳ ∧ '

(↓ ⊂*⎕↓ ⌈?→ ⍴~⊃⊃↓↓∪↓∪ ○⌈ *⌈⍴(↓⊤○⌈⍺ *↑↑ ?~⊤ ⌊⊤?;↑↓∘⍴' ○⌈∪↓↓∪
(↓ ⍴?∘↓→○∘↓⍴ ∩↓↓↑ (↓ ⊂*⎕↓ ⌈?→ ⊃?∘⌊↑↓→↓↑⍳ *⌈⍴(↓⊤↓∪ *⌈⍳ ?∩
→⊂↓∘' →⊂↓ *⌈⍴(↓⊤⍴ (↓ ⊂*⎕↓ ∩?~⌈∪ ⍴↓⊤⎕↓ ?⌈↑⍳ →? ⊤*○⍴↓ * (⊂?↑↓
⍴↓→ ?∩ ⌈↓(_~↓⍴→○?⌈⍴' ○⌈ ⍴?∘↓ (*⍳⍴ (↓ ∩↓↓↑ →⊂*→ (↓ *⊤↓ *⍴
⊃?⌈∩~⍴↓∪ *⍴ ↓⎕↓⊤ ;~→ (↓ ;↓↑○↓⎕↓ (↓ *⊤↓ ⊃?⌈∩~⍴↓∪ ?⌈ * ∘~⊃⊂
⊂○⍺⊂↓⊤ ↑↓⎕↓↑ *⌈∪ *;?~→ ∘?⊤↓ ○∘⌊?⊤→*⌈→ _~↓⍴→○?⌈⍴'
→⊂↓⊤↓*∩→↓⊤ ⊤↓*↑ ∘*→⊂↓∘*→○⊃⍴ (*⍴ ⌈?→ ?⌈↑⍳ ⊂○⍴ ⍴↓⊤○?~⍴ ?⊃⊃~⌊*→○?⌈
;~→ ⊂○⍴ ∩*⍴⊃○⌈*→○⌈⍺ ∪↓↑○⍺⊂→' *⍴⍵↓∪ ⍴?∘↓ ⍳↓*⊤⍴ ↑*→↓⊤ ⊂?(⊂↓
⊂*∪ ∘*⌈*⍺↓∪ →? ∩?⊤⍺↓ *⊂↓*∪ ⍴? ⊤*⌊○∪↑⍳ →? →⊂↓ ∩⊤?⌈→ ⊤*⌈⍵ ⊂↓
⊤↓⌊↑○↓∪ ;⍳ ⍴→~∪⍳○⌈⍺ →⊂↓ ∘*⍴→↓⊤⍴ ⌈?→ →⊂↓○⊤ ⌊~⌊○↑⍴' * ⌊⊤↓⍴⊃⊤○⌊→○?⌈
⍴?∘↓ ⌊?⌊~↑*⊤ (⊤○→↓⊤⍴ ?∩ →↓\→;??⍵⍴ ∘○⍺⊂→ ∪? (↓↑↑ →? ∘↓⌈→○?⌈
?⌈ →⊂↓○⊤ ⌊⊤↓∩*⊃↓⍴ *⍴ *⌈ *⌈→○∪?→↓ →? →⊂↓ ⌊?○⍴?⌈?~⍴ ∘↓∪○?⊃⊤○→⍳
?∩ →⊂↓○⊤ ~⌈○⌈⍴⌊○⊤↓∪ ⌊↓∪*⍺?⍺○⊃⍴'

APPENDIX F

FORMULA USED IN COMPUTING ARC SCORES

$$ARC = \frac{\#R - CR}{MR - CR}$$

Where

$$CR = \frac{\sum_{i}^{k} n_i}{N} - 1$$

$\#R$ = number of times a category item follows an item from the same category

$MR = N - k$

k = number of categories

N = number of items recalled

n_i = number of items recalled from category I

APPENDIX G

STEPWISE DISCRIMINANT FUNCTION ANALYSIS

A stepwise discriminant function analysis of the recall data from Experiment I was used in an endeavor to give some indication of the effects upon recall of the nonmanipulated structural variables in relation to the manipulated variable, mean distance between concept pairs. In this appendix, I describe stepwise discriminant function analysis and its application in this study.

Stepwise discriminant function analysis is a multivariate technique, based on a multiple regression model, for deriving, in a stepwise fashion, a discriminant function which maximally distinguishes between a set of n groups. Typically, one has scores on a number of variables for each S in each of the n groups and wishes to see if upon the bases of these variables a discriminant function can be formed which distinguishes between groups.

A program devised at the University of California, BMD07M (Dixon*, 1970), was used to perform the stepwise multiple discriminant function analyses used in this study. BMD07M performs 1-way analysis of variances on each predictor variable which might potentially enter into the discriminant function based on the criterion variable which distinguishes the n groups. The program then proceeds, in a stepwise fashion, to select, by means of a covariance analysis, a series of predictor variables to enter the discriminant function. The choice of variables to enter at each step is dictated by an F value which reflects the importance of that

* Biomedical Computer Programs, W. J. Dixon, editor. University of California Press, Los Angeles, California, 1970.

variable in discriminating between the groups after partialling out the variance accounted for by variables already in the discriminant function. Thus, from this analysis one can ascertain from two F values, independent F and F to remove, the independent importance of each predictor variable in distinguishing between the groups and because of the stepwise selection procedure the relative importance of the predictor variables for discriminating between groups. The program allows, among other things, for classification matrices to be computed at any step in the derivation of the discriminant function so that the classification accuracy of the function at that step can be evaluated.

APPENDIX H

METHODS OF COMPUTING MEASURES OF CONSISTENCY OF DEFINITION

The mean percentage of agreement in definition within Ss was computed as follows, with C_{ijt} denoting the set of concepts used by the i-th S to define concept j at time t:

$$\frac{\sum_{i=1}^{10}\left(\frac{\sum_{j=1}^{51}\left(\frac{\left|\bigcap_{t=1}^{2} C_{ijt}\right|}{\left|\bigcup_{t=1}^{2} C_{ijt}\right|}\right)}{51}\right)}{10} \times 100 \quad .$$

The first method of computing the mean percentage of agreement in definition across Ss was as follows:

$$\frac{\sum_{i=1}^{51}\left(\frac{\left|\bigcap_{i=1}^{10} C_{ij}\right|}{\left|\bigcup_{i=1}^{10} C_{ij}\right|}\right)}{51} \times 100 \quad .$$

The second method of computing the mean percentage of agreement in definition across Ss was as follows:

$$\frac{\sum_{j=1}^{51}\left(\frac{\sum_{k=1}^{9}\ \sum_{m=k+1}^{10}\left(\frac{\left|\bigcap_{i=k}^{m} C_{ij}\right|}{\left|\bigcup_{i=k}^{m} C_{ij}\right|}\right)}{45}\right)}{51} \times 100 \quad .$$

BIOGRAPHICAL SKETCH

James Davidson Hollan was born on October 31, 1945, in Dayton, Ohio. He graduated from Northeast High School in St. Petersburgh, Florida and, in 1969, completed a B.A. in psychology at the University of Florida. In 1971 he was awarded an M.S. in psychology from the University of Florida.

Since 1971 he has been concurrently pursuing a Ph.D. degree in psychology and a M.S. degree in electrical engineering. Upon completion of his education he will be employed by Clarkson College of Technology in Potsdam, New York.

J. D. Hollan is married to Carol A. Bowden of St. Petersburgh, Florida.

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

Henry S. Pennypacker, Chairman
Professor of Psychology

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

Wilse B. Webb, Cochairman
Graduate Research Professor of Psychology

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

Franz R. Epting
Associate Professor of Psychology

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

Nathan W. Perry
Professor of Psychology

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

Frank D. Vickers

Frank D. Vickers
Associate Professor of Computer and Information Sciences

This dissertation was submitted to the Department of Psychology in the College of Arts and Sciences and to the Graduate Council, and was accepted as partial fulfillment of the requirements for the degree of Doctor of Philosophy.

August, 1973

Dean, Graduate School